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The student should select 8 problems out of 10 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose at least one problem from each category listed above. Use one exam book (blue book) for each problem. Include your assigned number and NOT your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-S18/SPRING 2018
Ordinary Differential Equation #1

Solve the following ODE using Laplace transforms

\[ \dot{x} + 3x = t \cdot 1(t) \]

\[ x(0) = 1 \]
Ordinary Differential Equation #2

Solve the following initial value problem:

\[ \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0 \quad y(0) = 2, \quad \frac{dy}{dt}(0) = -3 \]
Partial Differential Equation #1

The edges of a thin plate are maintained at the temperatures shown in the figure below. Determine temperature distribution in the plate as the steady state. (Assume the large flat surfaces are insulated.)
Partial Differential Equation #2

Find the general solution of the equation

\[ e^z \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = ye^z \]
Linear Algebra #1

Find the eigenvalues and eigenvectors of the following matrix.

$$A_2 = \begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$
Find the matrix $X$ which satisfies the equation

$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} X = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$
Calculus #1

Compute: $\lim_{x \to \infty} \frac{x^2 + 3x - 10}{7x^2 - 5x + 4}$
Calculus #2

Evaluate the line integral

$$\oint_C [(x^3 - 2y)\,dx + (2y^4 + 4x)\,dy]$$

If $C$ is the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$
Numerical Methods #1

Consider the one-dimensional unsteady diffusion equation:

\[ \frac{\partial \phi}{\partial t} = \alpha \frac{\partial^2 \phi}{\partial x^2} \]

If we use the following transformation, the finite differencing equation is given by:

Step 1:
\[ \frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} = \alpha \frac{\phi_{i+1}^{n+1} - \phi_{i-1}^{n+1} - \phi_i^{n+1} + \phi_i^n}{\Delta x^2} \]

Step 2:
\[ \frac{\phi_i^{n+2} - \phi_i^{n+1}}{\Delta t} = \alpha \frac{\phi_{i+1}^{n+1} - \phi_{i-1}^{n+1} - \phi_i^{n+2} + \phi_i^{n+1}}{\Delta x^2} \]

In this method, Step 1 marches the solution from the bottom boundary (see Fig. below) to the top boundary. \((t,x)\) coordinate is transformed into numerical coordinate \((n,i)\).

(1) Analyze for stability using the Von Neumann Stability.
(2) What are the truncation errors?

[Hint] Von Neumann:

\[ f^n = V^n e^{i k \Delta x} \]

where \( k \) = wave number;

\( I = \sqrt{-1} \) = imaginary value; and

determine phase angle \( \theta = k \Delta x \)
Numerical Methods #2

Using the Euclidean norm, find the condition number of the following matrix.

\[ A = \begin{bmatrix} 0.0003 & 3 \\ 1 & 1 \end{bmatrix} \]
MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Thermal Science Stem

Open Book/Notes

February 2, 2018
Friday
1:00 pm – 5:00 pm
EMS E250

The problems are:

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Enter problem number(s) that you selected

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Fluid Mechanics #1

The water surface in a tank oscillates back and forth when you disturb the tank. The sloshing frequency $\Omega$ (Hz or 1/sec) can be expressed in a functional form $\Omega = f(L, h, g)$, where $L$ is the characteristic length (m) of tank, $h$ is the depth of water (m), and $g$ is the acceleration of gravity ($m/s^2$).

(a) Rewrite the equation in a dimensionless form.
(b) If a tank of water sloshes at 2.0 Hz on earth ($g \approx 9.8 \, m/s^2$), how fast would it oscillate on Mars ($g \approx 3.7 \, m/s^2$)?
Fluid Mechanics #2

Water flows from a very large tank through a 2 in. diameter tube. In the figure, the dark liquid in the manometer is mercury. Estimate the velocity in the pipe and the rate of discharge from the tank.
**Fluid Mechanics #3**

A circular arc gate ABC with the width (W) of 8 ft pivots about point O. For the position shown in the figure below, the radius of the gate is R = 4 ft and the angle \( \alpha = 45^\circ \).

Determine:

(a) The hydraulic force on the gate.
(b) Its line of action in the distance from O.
(c) The direction of the hydrostatic force.
**Thermodynamics #1**

A turbojet engine operates between pressure limits of 5 and 50 psia. The inlet air temperature to the compressor is -40°F and the upper temperature limit for the engine is 2000°F.

(1) Calculate the thrust for 1 lbm/sec of air flow, in lbf assuming isentropic compression and expansion and an inlet velocity of 300 ft/sec.

(2) Calculate the heat input per pound mass of air, in lbm.

(3) If the turbojet is moving with the velocity equal to the inlet velocity calculate the power developed by the thrust force, in hp.

In the above analyses assume constant specific heats. Also assume that the velocity at the exit of the turbine at point 4 is very small.
Thermodynamics #2

A gasoline engine has a volumetric compression ratio of 10 and before compression has air at 290K, 85 kPa, in the cylinder. The combustion peak pressure is 6000 kPa. Assume cold air properties. What is the highest temperature in the cycle? Find the temperature at the beginning of the exhaust (heat rejection) and the overall cycle efficiency.
**Thermodynamics #3**

Air expands through a turbine from 8 bar, 960 K to 1 bar, 450 K. The inlet velocity is small compared to the exit velocity of 90 m/s. The turbine operates at steady state and develops a power output of 2500 kW. Heat transfer between the turbine and its surroundings and potential energy effects are negligible. Modeling air as an ideal gas, determine (a) the mass flow rate of air in kg/s and (b) the exit area in m².
Heat Transfer #1

An engineer is designing a multiple-tube heating system that is placed in a duct with air supply to the building. He decides to carry out a preliminary test using one 2 cm copper tube carrying the condensed steam at 100°C. The velocity of air in the duct is 5 m/s with temperature of 20°C. The tube can be placed perpendicular to the flow, but because of the additional heat transfer surface area it may be advantageous to tilt the tube to the airflow. When the duct has a width of 1 m, determine the outcome of the planned tests.
Heat Transfer #2

A thermocouple junction is inserted in a large black duct to measure the temperature of hot gases flowing through the duct. The thermocouple junction is a 2-mm-diameter sphere with a surface emissivity of 0.60 and is placed in a gas stream moving at 3 m/s. If the thermocouple senses a temperature of $T_j = 320°C$ when the duct surface temperature is $T_s = 175°C$, what is the actual gas temperature, $T_g$ in °C if the hot gas has the following properties:

$v = 60.21 \times 10^{-6} \text{ m}^2/\text{s}, k = 0.0497 \text{ W/(m°C)}, \text{Pr} = 0.720, \text{and } (\mu/\mu_s) \sim 1.$
Heat Transfer #3

Consider a laminar steady Couette flow between two plates horizontally placed where there is a constant pressure gradient, \( dP/dx \). The top plate is moving at a constant speed of \( U_0 \) with the fixed temperature, \( T_0 \), while the bottom plate is fixed with a constant temperature, \( T_b \). The density, viscosity, and the thermal conductivity of the fluid are, \( \rho \), \( \mu \), and \( k \), respectively.

(1) Obtain the expression of the velocity profile. Let \( dP/dx = \text{Constant} \).
(2) Obtain the expression of the temperature profile when \( dP/dx = 0 \). It is suggested to include the dissipation term.
MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Machine Design Stem

Open Book/Notes

February 2, 2018
Friday
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QualExam-S18/SPRING 2018
Three identical, solid circular rods, each of radius ‘r’ and length ‘L’, are placed together to form a compression member (see the cross section shown in the figure). Assuming pinned-end conditions, determine the critical load as follows: (a) The rods act independently as individual columns, and (b) the rods are bonded by epoxy throughout their lengths so that they function as a single member. What is the effect on the critical load when the rods act as a single member?
Machine Design #2

For the linkage shown below, find its limit (toggle) positions in terms of the angle of link $O_2A$ referenced to the line of centers $O_2O_4$ when driven from link $O_2A$.
Machine Design #3

A 75 kg person stands at the free end of a diving board a total of 4.8 m long. The board can be modeled as a solid rectangular cross section 0.5 m wide and 4 cm thick with an effective modulus of elasticity of 20 GPa. The board is supported by a hinge at its root and a roller that can be moved along its length to adjust the reaction of the board. If the person deflects the tip of the board 0.15 m determine the distance from the hinge to the roller.
Kinematics & Dynamics #1

The double inclined plane supports two blocks A and B, each having a weight of 10 lb. If the coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.1$, determine the acceleration of each block.
The link rotates with a constant angular speed, $\omega$. The acceleration of point A has magnitude 1.118 $\text{m/sec}^2$ and its orientation angle is $\phi=26.57^0$ relative to the horizontal line. The acceleration of point B is vertical.

Calculate the angular speed, $\omega$, the magnitude of the acceleration of point B.
Kinematics & Dynamics #3

A pendulum consists of a 10kg mass at the end of a 2 m cable. Released from horizontal, determine the maximum tension that occurs in the cable, when the pendulum is at its lowest point.
Two solid cylinders of radii $R$ and masses $m$ rolling with no slip upon a surface are connected (through joints at their centers) to the walls and between themselves by springs with the spring constants of $k$, $k/2$, and $k$. Find natural frequencies and mode shapes in the general case, and for $R=0.1$ m, $m=10$ kg, $k=100$ N/m.
A large disk is attached to the end of a steel rod 5 mm in diameter and 2 m long with a shear modulus (G) of 80 GPa. The disk is rotated and released causing it to go through 10 oscillations in 30.2 seconds. Determine the polar mass moment of inertia of the disk.
Controls & Vibration #3

Consider the system shown in figure below, determine the value of $k$ such that the damping ratio ($\zeta$) is 0.5. Then obtain the rise time $t_r$, peak time $t_p$, maximum overshoot $M_p$, and settling time $t_s$ in the unit-step response. Find the range of $k$ that keeps the system stable.