MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part I
Mathematics and Fundamentals

Closed Book/Closed Notes

September 13, 2018
Thursday
1:00 pm - 5:00 pm
EMS E250

The problems are:		Enter problem number(s) that you selected	
Ordinary Differential Equations	2 problems	( )	( )
Partial Differential Equations	2 problems	( )	( )
Linear Algebra	2 problems	( )	( )
Calculus	2 problems	( )	( )
Numerical Methods	2 problems	( )	( )

The student should select 8 problems out of 10 problems given here. Write the problem numbers that you selected in (   ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your assigned number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-F18/FALL 2018

Ordinary Differential Equation #1

Solve the following ordinary differential equation.

$$y' + y \tan x = \sin 2x$$
$$y(0) = 1$$

Ordinary Differential Equation #2

Solve the following boundary-value problem, given '*y*' is a function of '*x*'.

$y'' + 2y' + y = 0 \qquad y(0) = 1 \qquad y(1) = 3$

Partial Differential Equation #1

Show that the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ has a general solution $u(x, t) = \Phi(x - ct) + \Psi(x + ct)$, where Φ and Ψ are arbitrary functions. Use the following coordinate transformation: $\xi = x + ct$ and $\eta = x - ct$.

Partial Differential Equation #2

The differential equation to describe the transient heat conduction is given as:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \qquad 0 \le \mathrm{x} \le 1$$

with the initial conditions of *u* is given by

I.C.:

$$u = 1 - x \qquad 0 \le \mathrm{x} \le 1$$

Solve the above equation for the boundary condition:

B.C.:

$$u = 0 \qquad \text{at } \mathrm{x} = 0$$

and

$$\frac{\partial \mathrm{u}}{\partial \mathrm{x}} = 0 \qquad \text{at } \mathrm{x} = 1$$

Data:

Thermal diffusivity: α=constant=1

The units are all dimensionless.

Linear Algebra #1

Let $$A = \begin{pmatrix} 2 & 1 - i \\ 1 + i & 1 \end{pmatrix}$$

Find a unitary matrix U that diagonalizes A.

Linear Algebra #2

Find the eigenvalues and corresponding eigenvectors for the matrix:

$$\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$$

Calculus #1

Given the parametric equations

$$x(t) = t^2 + 3$$
$$y(t) = \sin(t)$$

Find $\dfrac{d^2y}{dx^2}$

Calculus #2

Verify the Green's Theorem in the normal form for the field **F**= x **i** + y **j** and curve C that consists of the upper half of the unit circle and the X-axis interval [-1. 1].

Numerical Methods #1

We consider the Forward-Time and Centered Space method for the entire advection-diffusion equation

$$\frac{\partial \phi}{\partial t} = - u \frac{\partial \phi}{\partial x} + \alpha \frac{\partial^2 \phi}{\partial x^2} \qquad (1)$$

Transformation of PDE into FTCS gives

$$\phi_i^{n+1} = \phi_i^n - \frac{C}{2}\left(\phi_{i+1}^n - \phi_{i-1}^n\right) + d\left(\phi_{i+1}^n + \phi_{i-1}^n - 2\phi_i^n\right) \qquad (2)$$

Where

$$C = \frac{u \Delta t}{\Delta x}$$

$$d = \frac{\alpha \Delta t}{\Delta x^2}$$

Using the von Neumann stability analysis, analyze for stability.

[Hint] Von Neumann

$f_i^n = V^n e^{Iki\Delta x}$ where k = wave number;

$I = \sqrt{-1}$ = imaginary value; and

use phase angle $\theta = k\Delta x$

Numerical Methods #2

Using Newton Raphson method, find the first two iteration values of the root for

$$y = 3x^4 + 7x^2 + 6x + 1$$

Assume the initial guess is $x = 0$.

MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Thermal Science Stem

Open Book/Closed Notes
(Only books can be used, but no other materials)

September 14, 2018
Friday
1:00 pm – 5:00 pm
EMS E250

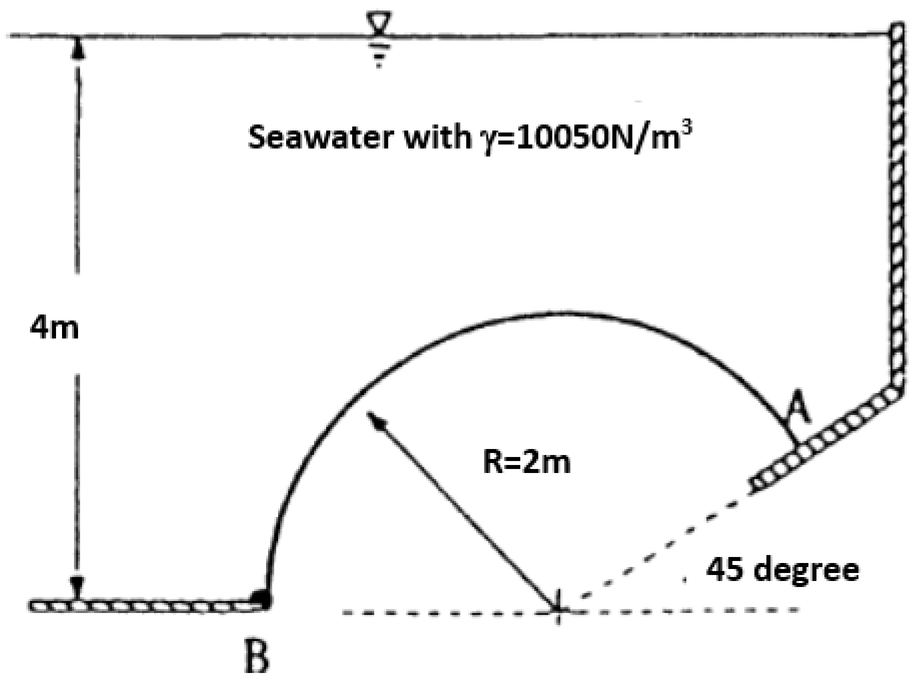
The problems are:		Enter problem number(s) that you selected
Fluid Mechanics	3 problems	( ) ( ) ( )
Thermodynamics	3 problems	( ) ( ) ( )
Heat Transfer	3 problems	( ) ( ) ( )

The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in (   ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your assigned number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

Fluid Mechanics #1

A gate is placed at the bottom of a tank filled with seawater with the specific weight of γ=10050 N/m^3. The gate AB is a 3 8th circle (R=2m) with the width of W=3m into the paper. The gate is hinged at B and resting on a wall surface at A.

(1) Compute the horizontal force on the gate.
(2) Compute the vertical force on the gate.
(3) Compute the reaction forces at A.
(4) Compute the vertical component reaction forces at B.
(5) Compute the horizontal component reaction forces at B.

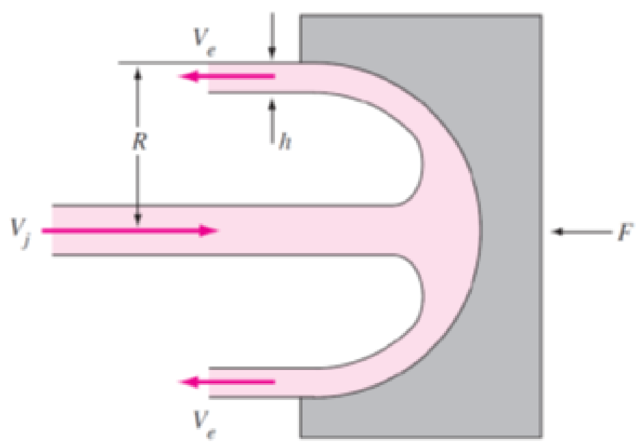


Fluid Mechanics #2

Suppose a water jet impinges on a cup cavity at velocity V_j as shown in the figure below. The water is turned 180° and exits, due to friction, at lower velocity, V_e (Looking from the left, the exit jet is a circular annulus of outer radius R and thickness h- , flowing toward the viewer.)

(a) Find the thickness h of the exit jet.

(b) Find the force F required to hold the cupped object in place.



Fluid Mechanics #3

A hot air balloon is to be used to lift a scientific payload into the sky. The balloon can be approximated as a sphere with a diameter of 10 m. The mass of the balloon and payload is 300 kg. Considering the balloon to be at sea level with the atmospheric conditions being at standard temperature and pressure, what does the temperature of the air inside the balloon need to be heated to in order to lift the balloon and payload into the air?

Thermodynamics #1

A tank initially containing air at 30 atm and 282 C is connected to a small turbine. Air discharges from the tank through the turbine, which produces work in the amount of 105.5 kJ. The pressure in the tank falls to 3 atm during the process and the turbine exhausts to the atmosphere at 1 atm. Employing the ideal gas model for the air with $k = 1.4$ and ignoring irreversibilities within the tank and the turbine, determine the volume of the tank, in L. Heat transfer with the atmosphere and changes in kinetic and potential energy are negligible.

Thermodynamics #2

You are asked to consider replacing an old air compressor in a factory with a new, more efficient compressor, but need to determine the energy savings for switching. Currently, the factory needs 2 kg/s of compressed air. The air intake to the compressor is 100 kPa and 15℃. The current compressor produces compressed air at 400 kPa and a temperature of 240℃. You find a new compressor which will produce the compressed air at 400 kPa with an isentropic efficiency of 80%. How much less power does the new compressor use than the old compressor?

Thermodynamics #3

Water is the working fluid in a Carnot vapor power cycle. Saturated vapor enters the turbine at (1) and saturated liquid enters the boiler at a pressure of 10 MPa at (4). The condenser pressure is 6 kPa from (2) to (3). Both turbine and pump are assumed to be isentropic.

Determine:

(a) The thermal efficiency.

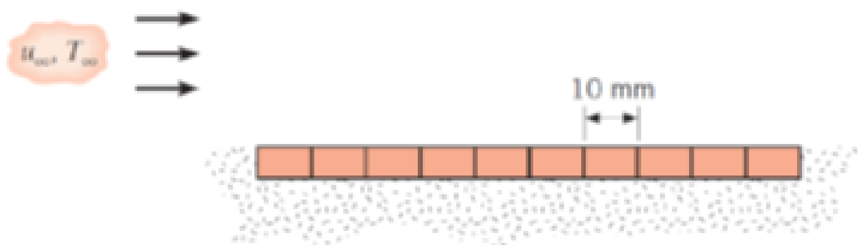
(b) The back work ratio.

(c) The heat transfer rate to the working fluid per unit mass passing through the boiler, in kJ/kg.

(d) The heat transfer rate from the working fluid per unit mass passing through the condenser, in kJ/kg.

Heat Transfer #1

An array of 10 silicon chips, each of length L = 10 mm on a side, is insulated on one surface and cooled on the opposite surface by atmospheric air in parallel flow with T = 24℃ and u_∞= 40 m/s. When in use, the same electrical power is dissipated in each chip, maintaining a uniform heat flux over the entire cooled surface. If the temperature of each chip may not exceed 80℃, what is the maximum allowable power per chip?



Heat Transfer #2

Air at 150℃ and 6 m/s enters a 5-m-long, thin-walled tube of 25-mm diameter. If steam at 20 bar and T_{sat} = 212℃ condenses on the outer surface, determine

(a) The outlet temperature of the air.
(b) The rate of heat transfer to the air.

Assume air properties are $\rho = 0.7740$ kg/m^3, $c_p = 1021$ J/(kg°C) $v = 32.13 \times 10^{-6}$ m^2/s, $k = 0.0373$ W/(m°C), and Pr = 0.686.

Heat Transfer #3

An engineer is designing a multiple-tube heating system that is placed in a duct with air supply to the building. He decides to carry out a preliminary test using one 2 cm copper tube carrying the condensed steam at 100°C. The velocity of air in the duct is 5 m/s with temperature of 20°C. The tube can be placed perpendicular to the flow, but because of the additional heat transfer surface area it may be advantageous to tilt the tube to the airflow. When the duct has width of 1 m, determine the outcome of the planned tests.

MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Machine Design Stem

Open Book/Closed Notes
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September 14, 2018
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EMS E250

The problems are:		Enter problem number(s) that you selected
Machine Design	3 problems	( ) ( ) ( )
Kinematics & Dynamics	3 problems	( ) ( ) ( )
Controls & Vibration	3 problems	( ) ( ) ( )

The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your student ID number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

Machine Design #1

A 6061-T451 aluminum SCUBA tank is a cylinder measuring 8 inches in outer diameter, 7 inches in inner diameter. The scuba tank is pressurized to 3,000 psi. Determine the factor of safety against ultimate failure according to the von Mises criteria. The ultimate strength of the aluminum is S_u = 32,000 psi. Note: the critical point is on the interior wall of the SCUBA tank.

Machine Design #2

Given a fourbar linkage with the link lengths:

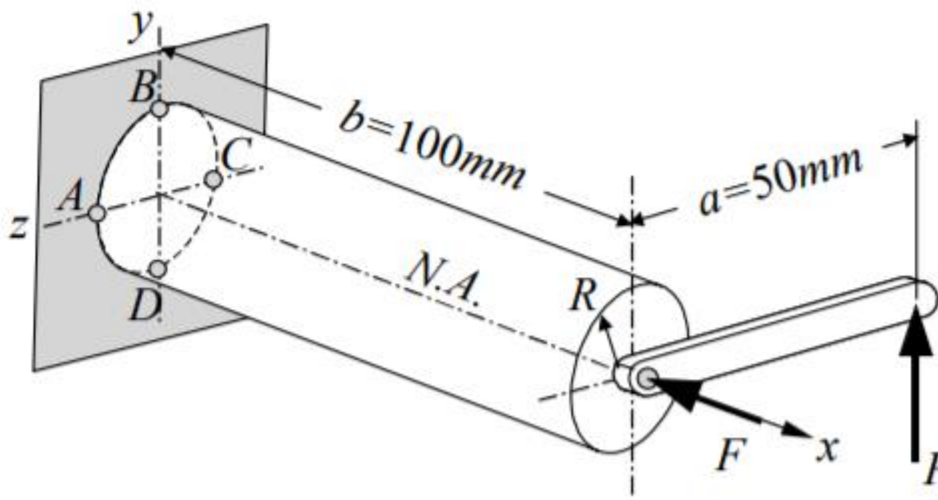
$L_1 = d = 100\ mm$
$L_2 = a = 40\ mm$
$L_3 = b = 120\ mm$
$L_4 = c = 80\ mm$

For $\theta_2 = 40°$, $\omega_2 = 25\ rad/sec$, $\theta_3 = 20.3°$ and $\theta_4 = 57.3°$

Find the values of ω_3, ω_4, V_A, V_{BA} and V_B.

Machine Design #3

Two forces P=18kN and F=15kN are applied to the shaft with a radius of R=20mm as shown. Determine the maximum normal and shear stresses developed in the shaft.

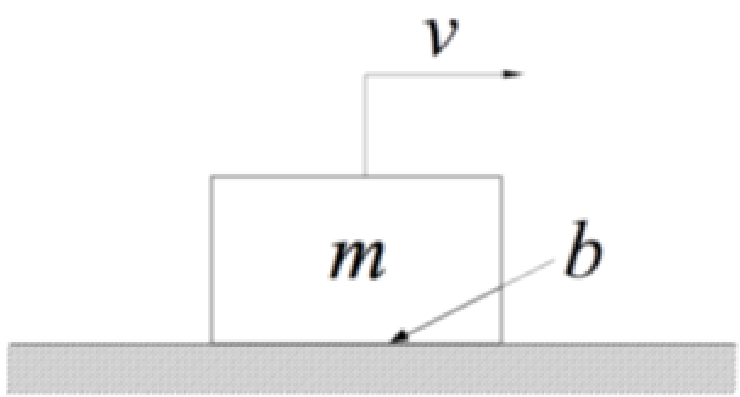


Kinematics & Dynamics #1

A car is going around a curve at 50 ft/s with a constant radius of 100 ft. Determine the necessary bank in the road such that there is no sideways force on the tires.

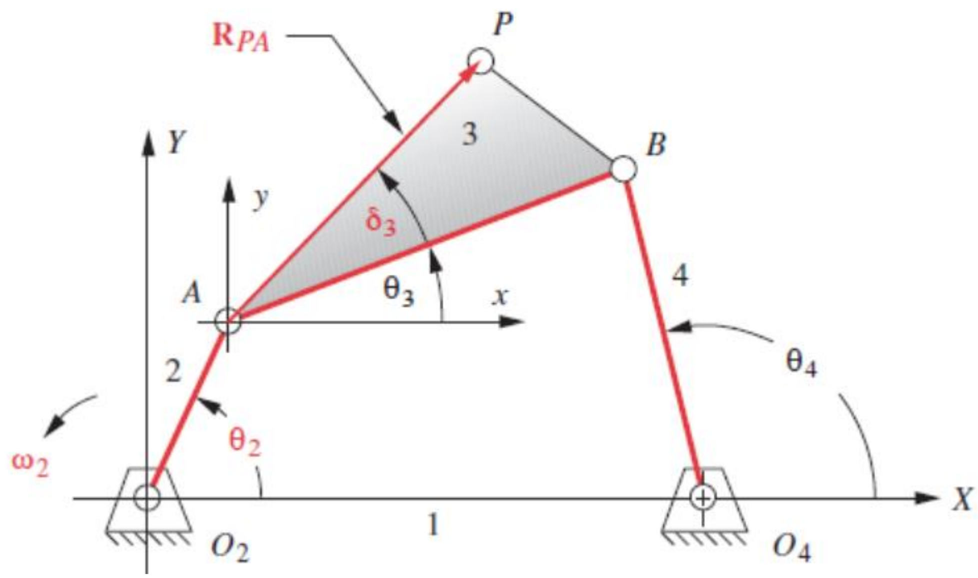
Kinematics & Dynamics #2

A mass m is moving over a plane with fluid friction coefficient b. The initial velocity is v_o. Find an expression for the time t for the velocity to drop to $v_o/2$.



Kinematics & Dynamics #3

Find the velocities of the pin joints A and B and the instant centers I_{13} and I_{24} using the graphical method. Find angular velocities of the links 3 and 4 and the velocity of the point P on the coupler.



Given:

Link lengths:

Link 1	$d := 6 \cdot in$	Link 2	$a := 2 \cdot in$
Link 3	$b := 7 \cdot in$	Link 4	$c := 9 \cdot in$

Crank Angle: $\theta_2 := 30 \cdot deg$

Crank Velocity: $\omega_2 := 10 \cdot rad \cdot sec^{-1}$

Coupler point data:

$R_{pa} := 6 \cdot in$ $\delta_3 := 30 \cdot deg$

Controls & Vibration #1

The characteristic equation of a closed looped system is given by

$$C(s) = 1 + \frac{K}{s(s+1)(s+3)} = 0$$

Find the value of the gain K when the system is marginally stable. Where are the closed loop pole locations at this value of K.

Controls & Vibration #2

A uniform rigid bar carrying two springs, dampers and a mass is applied with a couple at hinged end (Point O). Assuming small angular deformation Θ

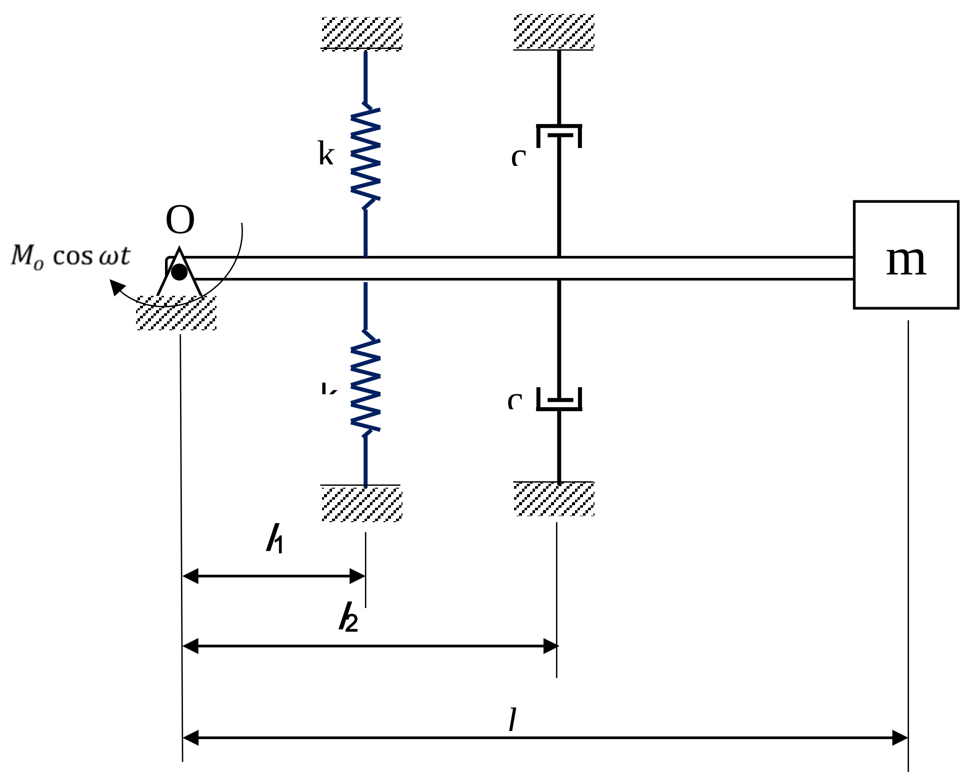
(a) Derive the equation of motion.

(b) Determine equivalent mass moment of inertia, spring stiffness, and damping coefficient if the system has following data:

$$l_1 = 0.25, l_2 = 0.5, l = 1, m = 40\ kg\ , J_0 = 5\ kgm^2,$$
$$k = 277\ KN/m, c = 2500\ Ns/m, M_o = 600\ Nm, \omega = 105\ rad/s$$

(c) Find total response if the system has following initial condition is

$$\Theta_o = 0.01\ rad, and\ \dot{\Theta}_o = 0$$



Controls & Vibration #3

Two solid cylinders of radii R and masses m rolling with no slip upon a surface are connected (through joints at their centers) to the walls and between themselves by springs with the spring constants of k, k/2, and k. Find natural frequencies and mode shapes in the general case, and for R=0.1 m, m=10 kg, k=100 N/m.