

THERE'S SO MUCH OUT  
THERE! HOW DO I FIND A  
GOOD MATHEMATICAL  
TASK?

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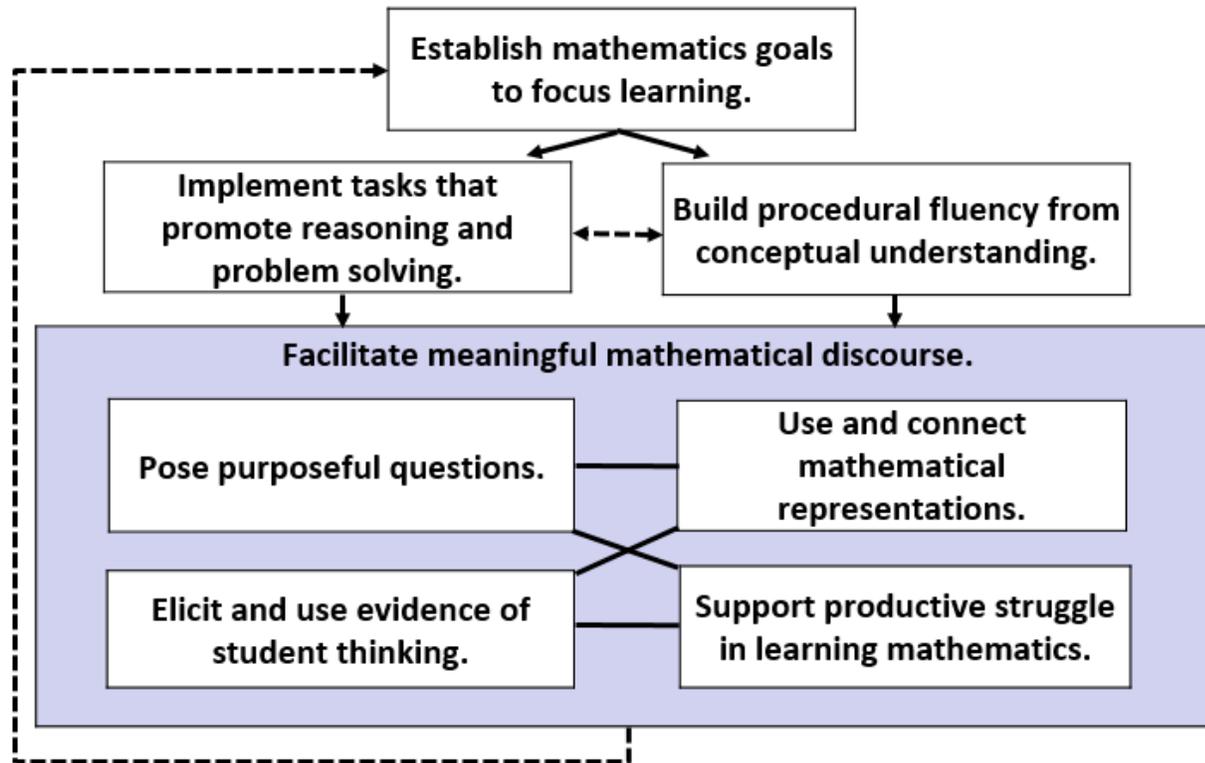
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# SESSION AGENDA

- What do you do now?
- Levels of Cognitive Demand
- Examining tasks
- Task implementation and modification

# WHY ARE INSTRUCTIONAL TASKS IMPORTANT?



(NCTM, 2017, p. 215)

# WHAT DO YOU DO NOW?

- What sources do you use for finding materials to use in the classes you teach?
- What do you look for when you're looking for a lesson or mathematical task?
  - Does cost matter?
  - How do you judge the quality of a lesson or task?

# COMPARING TWO TASKS

- Think privately about how you would go about solving each task.
- Then, turn and talk with someone near you.

# COMPARING TWO TASKS

The manager of a movie theater found that Saturday's sales were \$3675.

He knew that a total of 650 tickets were sold Saturday.

Adult tickets cost \$7.50, and children's tickets cost \$4.50.

How many of each kind of ticket were sold?

Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it. She said,

***I wonder whether the dollar belongs inside the cash box or not.***

The price of tickets for the dance was 1 ticket for \$5 (for individuals) or 2 tickets for \$8 (for couples). She looked inside the cash box and found \$200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?

# SIMILARITIES AND DIFFERENCES?

How are the tasks the same and how are they different?

## **SIMILARITIES**

- Both involve systems of equations
- Both require some knowledge of how to solve equations

## **DIFFERENCES**

- The amount of thinking and reasoning required
- The number of ways the problem can be solved
- Possible solution strategies
- The range of ways to enter the problem

# MATHEMATICAL TASKS

Not all tasks are created equal, and ***different tasks will provoke different levels and kinds of student thinking.***

Stein, Smith, Henningsen, & Silver, 2009

The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

# LEVELS OF COGNITIVE DEMAND

- Low Level Tasks



- Memorization



- Procedures without Connections

- High Level Tasks



- Procedures with Connections



- Doing Mathematics

- Require complex and nonalgorithmic thinking – a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the tasks and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive efforts and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

# CHARACTERIZING MATHEMATICAL TASKS

- Consider the tasks in the task packet.
  - Note: These tasks and lessons were collected from a number of popular online sources
- In pairs, decide how would you classify each task with respect to cognitive demand. Be prepared to explain why you made this classification.

# CSI: The Real Number System



Detectives,

A thief, working under the alias Guapo Arcsin, has caused a ruckus all throughout Central America. It is believed that Guapo is a member of the evil genius group, the Mathemagicians. From what the detectives have gathered thus far from the previous thefts, the Mathemagicians are building a world conquering device.

Fortunately, Guapo has left behind a trail of notes and a cryptic text message that he has told us will calculate toward his favorite number. Thus far there are six suspects that police have questioned. It is hoped that someone with a relatively strong number sense can crack some codes that have puzzled the detectives on the case so far.

Your job is to bring Guapo to justice and save the planet. You need to be prepared to state your case and demonstrate your understanding of the following skills that Guapo is known to use in his notes.



- The Real Number System
- Properties of Real Numbers

## Mixture Word Problems

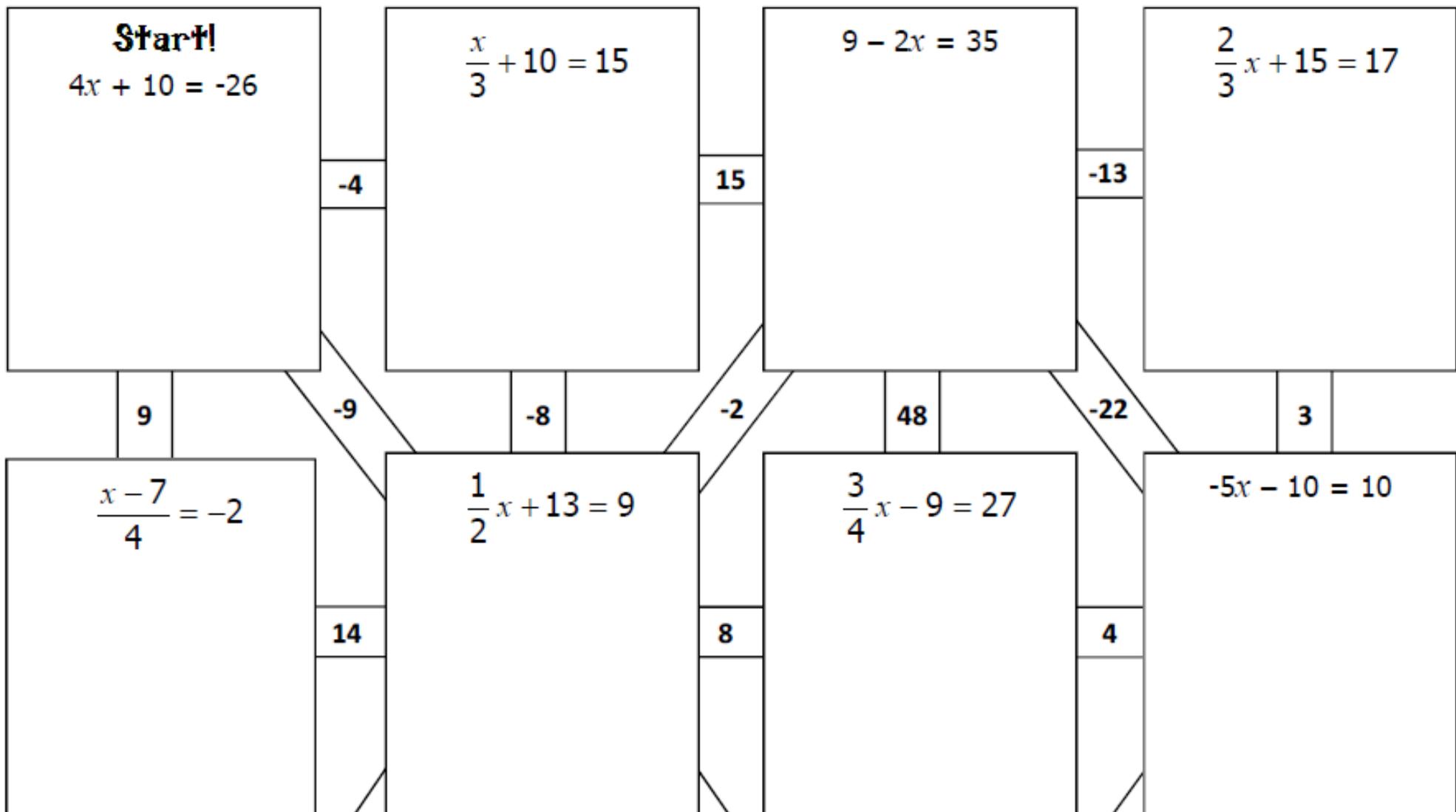
Name \_\_\_\_\_

Date \_\_\_\_\_ Period \_\_\_\_\_

- 1)  $2 \text{ m}^3$  of soil containing 35% sand was mixed into  $6 \text{ m}^3$  of soil containing 15% sand. What is the sand content of the mixture?
- 2) 9 lbs. of mixed nuts containing 55% peanuts were mixed with 6 lbs. of another kind of mixed nuts that contain 40% peanuts. What percent of the new mixture is peanuts?
- 3) 5 fl. oz. of a 2% alcohol solution was mixed with 11 fl. oz. of a 66% alcohol solution. Find the concentration of the new mixture.
- 4) 16 lb of Brand M Cinnamon was made by combining 12 lb of Indonesian cinnamon which costs \$19/lb with 4 lb of Thai cinnamon which costs \$11/lb. Find the cost per lb of the mixture.

# two-step eQuATion MAZE!

Directions: Use your solutions to navigate through the puzzle. **SHOW ALL STEPS!!!!**



Name: \_\_\_\_\_

Block: \_\_\_\_\_

Date: \_\_\_\_\_

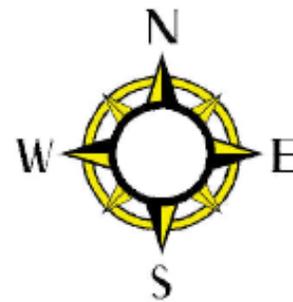
## Geometry Project

## Fencing Your Property

You want to put a fence around your large yard. There are two companies that you have found to do the work. They have each given you a quote for how much the work will cost. Of course, you want to find out which company will be the cheapest.

The boundary of your yard is determined by five trees. The lines connecting them form the edge of your property. Shown below are the descriptions for the positions of the trees relative to your house.

TREE	Position (relative to your house)
1	100 ft. east
2	40 ft east, 80 ft south
3	40 ft west, 120 ft south
4	90 ft west, 60 ft north
5	20 ft east, 110 ft north



**STEP 1:** On graph paper, mark the position of each of the trees on your land. Let each block of the graph paper represent a 10-foot by 10-foot square. Using a straightedge, connect Tree 1 to Tree 2, Tree 2 to Tree 3, Tree 3 to Tree 4, and so on.

# The Devil and Daniel Webster

NAME \_\_\_\_\_

The devil made a proposition to Daniel Webster. The devil proposed paying Daniel in the following way:

*On the first day, I will pay you \$1000 early in the morning. At the end of the first day, you must pay me a commission of \$100; so, your net salary that day is \$900. At the start of the second day, I will double your amount left at the end of the previous day. So, at the beginning of the second day, I will pay you \$1800; but at the end of the second day, you must double the amount that you pay me to \$200. Will you work for me for a month?*

1. After reading the salary proposal, would you work for the devil for a month? \_\_\_\_\_
2. Complete the following chart:

DAY	SALARY FOR DANIEL WEBSTER	DEVIL'S COMMISSION	MONEY AT THE END OF THE DAY
1	\$1000	\$100	\$900
2	\$1800		
3			
4			

# Evaluating Exponential Expressions

Henry explains why  $4^{3/2} = 8$ :

*I know that  $4^3$  is 64 and the square root of 64 is 8.*

Here is Henrietta's explanation for why  $4^{3/2} = 8$ :

*I know that  $\sqrt{4} = 2$  and the cube of 2 is 8.*

# OUT OF LEFT FIELD

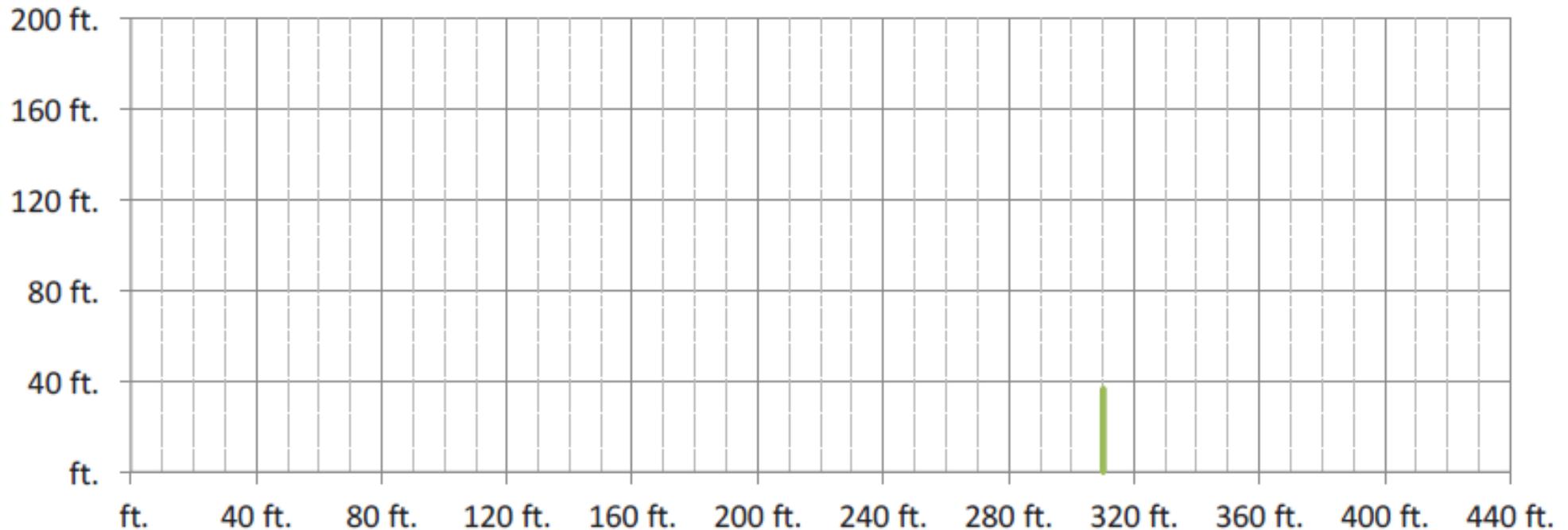
From which MLB ballpark is it hardest to hit a home run?

name

date

## Act One: Going Yard

- 1 The left field wall in Boston's Fenway Park – aka **Green Monster** – is 310 feet from home plate and 37 feet tall. The following equations represent three hypothetical trajectories of a baseball hit down the left field line. Graph each trajectory, and use the graph to find the key points below. Will each hit be a home run?



# Choosing an appropriate growth model

Below are population estimates for the larger metropolitan areas of Paris (France), Shenzhen (China), and Lagos (Nigeria) for each decade between 1950 and 2010:

City	1950	1960	1970	1980	1990	2000	2010
Paris	6,300,000	7,400,000	8,200,000	8,700,000	9,300,000	9,700,000	10,500,000
Shenzhen	3100	8000	22,000	58,000	875,000	6,600,000	10,000,000
Lagos	330,000	760,000	1,400,000	2,600,000	4,800,000	7,300,000	11,000,000

- For each city, decide if the population data can be accurately modeled by a linear, quadratic, and/or exponential function. Explain.
- If you found one or more good models for a city population, what predictions would those models make for future decades? Are these reasonable?

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## Best Buy Tickets

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Susie is organizing the printing of tickets for a show her friends are producing. She has collected prices from several printers and these two seem to be the best.

### **SURE PRINT**

Ticket printing  
25 tickets for \$2

### **BEST PRINT**

Tickets printed  
\$10 setting up  
plus  
\$1 for 25 tickets

Susie wants to go for the best buy

She doesn't yet know how many people are going to come.

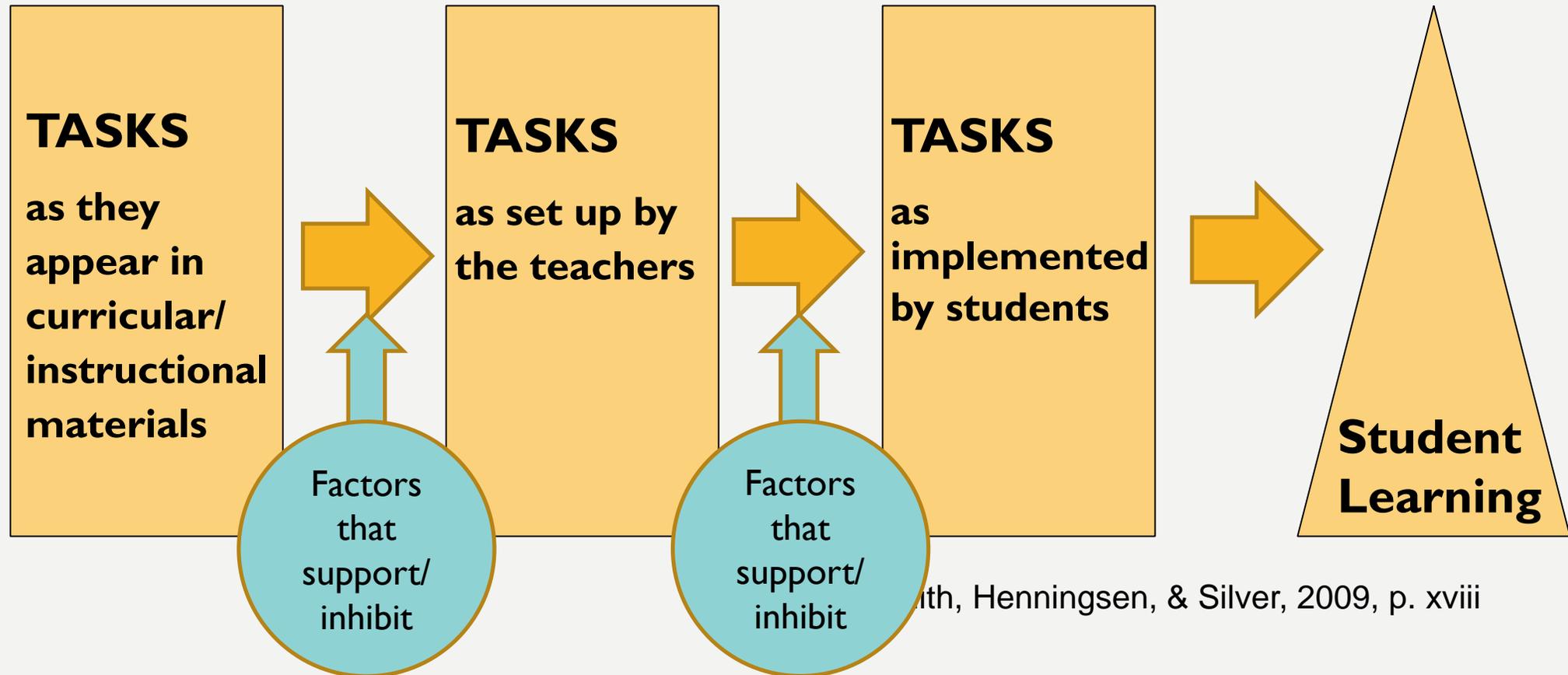
Show Susie a couple of ways in which she could make the right decision, whatever the number.

Illustrate your advice with a couple of examples.



# TASK FRAMEWORK

# TASK FRAMEWORK



with, Henningsen, & Silver, 2009, p. xviii

# FACTORS ASSOCIATED WITH THE DECLINE OF HIGH-LEVEL COGNITIVE DEMANDS

- Routinizing problematic aspects of the task
- Shifting the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer
- Providing insufficient time to wrestle with the demanding aspects of the task or so much time that students drift into off-task behavior
- Engaging in high-level cognitive activities is prevented due to classroom management problems
- Selecting a task that is inappropriate for a given group of students
- Failing to hold students accountable for high-level products or processes

# FACTORS ASSOCIATED WITH THE MAINTENANCE OF HIGH-LEVEL COGNITIVE DEMANDS

- Scaffolding of student thinking and reasoning
- Providing a means by which students can monitor their own progress
- Modeling of high-level performance by teacher or capable students
- Pressing for justifications, explanations, and/or meaning through questioning, comments, and/or feedback
- Selecting tasks that build on students' prior knowledge
- Drawing frequent conceptual connections
- Providing sufficient time to explore

# DOES MAINTAINING COGNITIVE DEMAND MATTER?

- The research shows that **maintaining the cognitive complexity of instructional tasks through the task enactment phase is associated with higher student achievement.**

Stein & Lane, 1996; Stigler & Hiebert, 2004; Boaler & Staples, 2008; and more



# MODIFYING TASKS

# STRATEGIES FOR INCREASING THE COGNITIVE DEMAND OF TASKS

- Ask students to create real-world stories for “naked number” problems.
- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, with a context).
- Use a task “out of sequence” before students have memorized a rule or have practiced a procedure that can be routinely applied.
- Eliminate components of the task that confine student thinking or provide too much scaffolding.
- Create opportunities for repeated reasoning or pattern finding
- Create a prompt that asks students to write about the meaning of the mathematics concept.
- Add a prompt that asks students to make note of a pattern or to make a mathematical conjecture and to test their conjecture.
- Include a prompt that requires students to make a generalization.
- Include a prompt that requires students to compare solution paths or mathematical relationships and write about the relationship between strategies or concepts.
- Select numbers carefully so students are more inclined to note relationships between quantities (e.g., two tables can be used to think about the solutions to the four, six, or eight tables).

# HOW MIGHT YOU MODIFY THIS TASK?

A local theater is showing an animated movie. They charge \$5 per ticket for a child and \$12 per ticket for an adult. They sell a total of 342 tickets and make a total of \$2550. We want to try to find out how many of each type of ticket they sold. Let  $c$  represent the number of children's tickets sold and  $a$  represent the number of adult tickets sold.

- (a) Write an equation that represents the fact that 342 tickets were sold.
- (b) Write an equation representing the fact that they made a total of \$2250.
- (c) Solve the system you created in (a) and (b) by the elimination method.

# ONE POSSIBLE MODIFICATION

The theater you run charges \$4 for child tickets and \$12 for adult tickets.

- (a) What's a large amount of money you could make?
- (b) What's a small amount of money you could make?
- (c) Your no-good kid brother is working the cash register. He told you he made:
  - \$2,550 on Friday
  - \$2,126 on Saturday
  - \$1,968 on Sunday

He's lying about at least one of those. Which ones? How do you know?

# ANOTHER POSSIBLE MODIFICATION

- A local theater is showing a popular animated movie. They charge admission for each person.
- The theater charges \$5 per ticket for a child and \$12 per ticket for an adult.
- They made a total of \$2550.
- They sold a total of 342 tickets.

# TASK MODIFICATION

- Choose one of the tasks you examined today.
- How could you modify the task to raise the level of cognitive demand?
- What messages do the differences in the tasks (original vs. modified) send students?



RESOURCES

# GOOD ONLINE RESOURCES

- Illustrative Mathematics
- Inside Mathematics
- Mathalicious
- Mathematics Assessment Project
- Achieve the Core
- Robert Kaplinsky's lessons
- Dan Meyer's Three-Act Tasks
- Achieve.org
- 101 Questions
- YouCubed
- From NCTM:
  - Illuminations
  - Reasoning and Sense Making Task Library
  - Activities with Rigor and Coherence
  - Problems of the Week

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