

5 PRACTICES

For Orchestrating Productive Mathematics Discussions

By Margaret S. Smith, Mary Kay Stein



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WHAT ARE THE 5 PRACTICES?



- 0 – Selecting a rich task with a high cognitive demand
- 1 – Anticipating student solutions
- 2 – Monitoring student responses while the students are working
- 3 – Selecting students to present their work during whole-class discussion
- 4 – Sequencing the responses that will be displayed in a specific order
- 5 – Connecting different student responses and connecting responses to key mathematical ideas.



Purpose -

- US math students are strong in less cognitively demanding tasks : ie solving equations, using formulas
- Have weaknesses in using real-world situations and interpreting the math of a real-world problem

Reasons-

- US students are given weak questions – low quality, fake contexts
- Strategies students learn are unhelpful in the real world (too much drill and kill)
- Students don't engage with mathematics the way mathematicians do

Mathematical Mindsets, Jo Boaler, 2016

RICH TASKS

- Have an entry point for every student (low floor)
- Have ways to extend the problem if students find an answer to the initial question (high ceiling).
- Real-life context isn't always necessary
- If the problem has a context it should be based in reality
- The problems should require some decision making for the students.

Boaler, 2016

RICH TASKS

Resources–

- There is a collection of resources at <https://www.mictm.org/index.php/resources/rich-math-tasks>
- www.youcubed.org
- <https://nrich.maths.org/8702>
- <http://www.estimated180.com/>

ZIOS AND ZEPTS

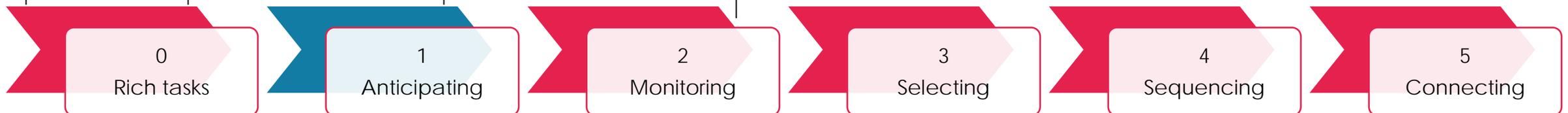
On the planet Vuv there are two sorts of creatures. The Zios have 3 legs and the Zepts have 7 legs. The great planetary explorer Nico, who first discovered the planet, saw a crowd of Zios and Zepts. Suddenly they all rolled over onto their backs and put their legs in the air. He counted 52 legs. How many Zios and how many Zepts were there?



WORK TIME

| Anticipated Solutions | Instructional Support | |
|-----------------------|--|---|
| | Assessing Questions | Advancing Questions |
| Draw a picture | <ul style="list-style-type: none"> • Can you tell me what you did? • How did you decide what to draw? • | <ul style="list-style-type: none"> • Is there more than one solution? • Can you represent your work in another manner? |
| Grouping | <ul style="list-style-type: none"> • Can you tell me what you did? • How did you decide to group your Zios and Zephs? • How could you explain your strategy to others? | <ul style="list-style-type: none"> • How could you write your method in “official” mathematical language? • Is there more than one solution? |
| Organized List | <ul style="list-style-type: none"> • Can you tell me what you did? • Why did you decide to create a list? • How did you decide which numbers went into your list? | <ul style="list-style-type: none"> • Can you write your thinking in another way? • Is there more than one possible solution to this problem? |
| Graph | <ul style="list-style-type: none"> • Can you tell me what you did? • How did you select your data points? • Why did you choose the locations/scale of your axis? | <ul style="list-style-type: none"> • Can you write an equation to fit your graph? • Is there more than one solution? |
| Equation | <ul style="list-style-type: none"> • Can you tell me what you did? • Why did you choose the variables that you used? • What do your numbers represent? • What do your variables represent? | <ul style="list-style-type: none"> • What more would you need to solve this equation? • Is there more than one solution? |
| Trial and Error | <ul style="list-style-type: none"> • Can you tell me what you did? • Why did you decide to take this path? | <ul style="list-style-type: none"> • Is there a way that you could arrive at your answer using more traditional math strategies? • Is there more than one solution? |
| Other | • | • |

- Before students start, I consider
 - Possible solution strategies
 - Questions to assess student understanding
 - Questions to advance student thinking.
- The questions are laid out on a monitoring chart





DISCUSSION

| Anticipated Solutions | Instructional Support | | Who and What |
|-----------------------|--|---|--------------|
| | Assessing Questions | Advancing Questions | |
| Draw a picture | <ul style="list-style-type: none"> Can you tell me what you did? How did you decide what to draw? | <ul style="list-style-type: none"> Is there more than one solution? Can you represent your work in another manner? | |
| Grouping | <ul style="list-style-type: none"> Can you tell me what you did? How did you decide to group your <i>Zios</i> and <i>Zepts</i>? How could you explain your strategy to others? | <ul style="list-style-type: none"> How could you write your method in "official" mathematical language? Is there more than one solution? | |
| Organized List | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to create a list? How did you decide which numbers went into your list? | <ul style="list-style-type: none"> Can you write your thinking in another way? Is there more than one possible solution to this problem? | |
| Graph | <ul style="list-style-type: none"> Can you tell me what you did? How did you select your data points? Why did you choose the locations/scale of your axis? | <ul style="list-style-type: none"> Can you write an equation to fit your graph? Is there more than one solution? | |
| Equation | <ul style="list-style-type: none"> Can you tell me what you did? Why did you choose the variables that you used? What do your numbers represent? What do your variables represent? | <ul style="list-style-type: none"> What more would you need to solve this equation? Is there more than one solution? | |
| Trial and Error | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to take this path? | <ul style="list-style-type: none"> Is there a way that you could arrive at your answer using more traditional math strategies? Is there more than one solution? | |
| Other | | | |

- While the students are working, I use my monitoring chart to help me remember which questions to ask without helping too much
- Productive struggle is vital to effective student work
- I note which groups are working and which solution strategies they are using.
- I refer back to these notes as the lesson continues.



| Anticipated Solutions | Instructional Support | | Who and What |
|-----------------------|--|---|--------------|
| | Assessing Questions | Advancing Questions | |
| Draw a picture | <ul style="list-style-type: none"> Can you tell me what you did? How did you decide what to draw? | <ul style="list-style-type: none"> Is there more than one solution? Can you represent your work in another manner? | |
| Grouping | <ul style="list-style-type: none"> Can you tell me what you did? How did you decide to group your <i>Zios</i> and <i>Zeps</i>? How could you explain your strategy to others? | <ul style="list-style-type: none"> How could you write your method in "official" mathematical language? Is there more than one solution? | |
| Organized List | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to create a list? How did you decide which numbers went into your list? | <ul style="list-style-type: none"> Can you write your thinking in another way? Is there more than one possible solution to this problem? | |
| Graph | <ul style="list-style-type: none"> Can you tell me what you did? How did you select your data points? Why did you choose the locations/scale of your axis? | <ul style="list-style-type: none"> Can you write an equation to fit your graph? Is there more than one solution? | |
| Equation | <ul style="list-style-type: none"> Can you tell me what you did? Why did you choose the variables that you used? What do your numbers represent? What do your variables represent? | <ul style="list-style-type: none"> What more would you need to solve this equation? Is there more than one solution? | |
| Trial and Error | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to take this path? | <ul style="list-style-type: none"> Is there a way that you could arrive at your answer using more traditional math strategies? Is there more than one solution? | |
| Other | | | |

- Keeping in mind my instructional goals, I select groups to present.
- I want the groups to highlight the objectives of the lesson or common misconceptions
- I do not want this to become a "show and tell" moment, so choosing groups judiciously is important.



| Anticipated Solutions | Instructional Support | | Who and What | Order |
|-----------------------|--|---|--------------|-------|
| | Assessing Questions | Advancing Questions | | |
| Draw a picture | <ul style="list-style-type: none"> Can you tell me what you did? How did you decide what to draw? | <ul style="list-style-type: none"> Is there more than one solution? Can you represent your work in another manner? | | |
| Grouping | <ul style="list-style-type: none"> Can you tell me what you did? How did you decide to group your <i>Zios</i> and <i>Zepts</i>? How could you explain your strategy to others? | <ul style="list-style-type: none"> How could you write your method in "official" mathematical language? Is there more than one solution? | | |
| Organized List | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to create a list? How did you decide which numbers went into your list? | <ul style="list-style-type: none"> Can you write your thinking in another way? Is there more than one possible solution to this problem? | | |
| Graph | <ul style="list-style-type: none"> Can you tell me what you did? How did you select your data points? Why did you choose the locations/scale of your axis? | <ul style="list-style-type: none"> Can you write an equation to fit your graph? Is there more than one solution? | | |
| Equation | <ul style="list-style-type: none"> Can you tell me what you did? Why did you choose the variables that you used? What do your numbers represent? What do your variables represent? | <ul style="list-style-type: none"> What more would you need to solve this equation? Is there more than one solution? | | |
| Trial and Error | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to take this path? | <ul style="list-style-type: none"> Is there a way that you could arrive at your answer using more traditional math strategies? Is there more than one solution? | | |
| Other | | | | |

- Choosing the order of the groups carefully will highlight what you want students to take away
- You can start from a partial solution and move towards a complete response
- You can start with the most common solution path and move towards less conventional strategies
- Be careful in that you don't always start with the least sophisticated strategy – it can be powerful, but it could backfire if used too often



- During the presentations, it is important that it not just be show and tell
 - Students need to be listening and respond to each other's presentations
 - The teacher needs to ask students to critique each other's understanding
- Asking the students to make the connections between the solution strategies and critiquing each other's thinking is a powerful way for them to solidify their own understanding of the mathematics.
- It is important that the teacher guides the conversation and doesn't say anything that a kid could say.
 - I've already made the connections – it's their turn now.





RESULTS OF CLASSROOM USE

Practice 0: Task Selection

The Task: Domino Effect

I chose the Domino Effect Problem from Mathalicious because it provides the students with multiple entry points, and was a good lead in to A.CED.1 writing equations to describe patterns.

Practice 1: Anticipating

During the course of my planning, I created this anticipation and monitoring chart to help me keep track of what students are doing and have questions prepared so that I am ready if they need extension or help getting started. I compiled anticipated solution strategies and questions to help move students along or advance their thinking.

| Anticipated Solutions Instructional Support | Who and What | |
|--|--|--|
| | Assessing Questions | Advancing Questions |
| Using Proportional reasoning. | <ul style="list-style-type: none"> Can you tell me what you did? What made you think to use proportions? | <ul style="list-style-type: none"> Would getting twice as many toppings make the pizza cost twice as much? Can you think of other situations where the price is not proportional to the number you buy? |
| Number Sense | <ul style="list-style-type: none"> Can you tell me what you did? How did you know that your strategy made sense? How could you explain your strategy to others? | <ul style="list-style-type: none"> How could you write your method in "official" mathematical language? If we approached a similar problem, is there a more efficient strategy you could apply? |
| Table | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to create a table? How did you decide which numbers went into your table? | <ul style="list-style-type: none"> Can you write your thinking in another way? Can you create another representation that describes the situation in your table? Can you predict how much a 5 or 6 topping pizza would cost? How would your table help? |
| Graph | <ul style="list-style-type: none"> Can you tell me what you did? How did you select your data points? Why did you choose the locations/scale of your axis? | <ul style="list-style-type: none"> Can you write an equation to fit your graph? Can you predict how much a 5 or 6 topping pizza would cost? How would your graph help? |
| Equation | <ul style="list-style-type: none"> Can you tell me what you did? Why did you choose the variables that you used? What do your numbers represent? What do your variables represent? | <ul style="list-style-type: none"> Can you write your thinking another way? Can you predict how much a 5 or 6 topping pizza would cost? How would your equation help? Can you create a graph of your equation? |
| Other | <ul style="list-style-type: none"> Can you tell me what you did? Why did you decide to take this path? | |

Here are the prices for a medium 2-topping pizza and a medium 4-topping pizza.
How much do you think Domino's charges per topping?
How much would you pay for a plain cheese pizza?

| | | |
|---|--|---------|
|  | Medium (12") Hand Tossed Pizza Whole: Pepperoni, Green Peppers | \$13.97 |
|  | Medium (12") Hand Tossed Pizza Whole: Bacon, Chicken, Mushrooms, Green Peppers | \$16.95 |

Practice 2: Monitoring

As students were working, I walked to each table and saw the different ways that the students were approaching the problem. I used my monitoring chart to ask them questions to move them forward in the task itself or move them ahead of the task.

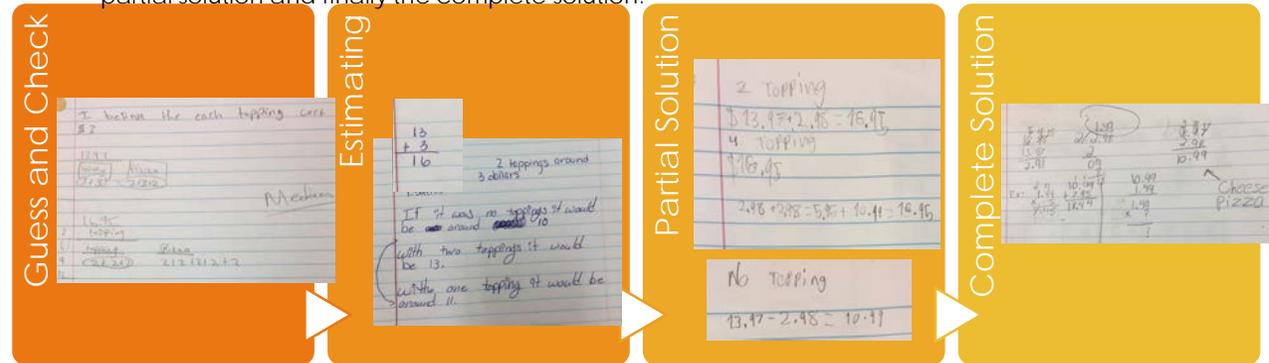
While I was watching, students approached the problems through guess and check, estimating, and using number sense.

Practice 3: Selecting

After seeing the approaches from each of my groups, I made careful selections of who I wanted to speak about the problems. I chose groups who had unique approaches and also groups who approached it the same way as many other groups but were able to explain their methods well.

Practice 4: Sequencing

I chose to have the group who used guess and check present first, followed by estimation, then the partial solution and finally the complete solution.



Practice 5: Connecting

I used my sequence to connect the thinking of each group. The guess and check group led to a conversation about how they were choosing topping costs, there were some misconceptions, but the thinking was solid. I followed that with the group who estimated, from there we were able to get more specific in our solution and the thinking behind it. My final group worked on the complete solution and

Student Thoughts

"It was a bit challenging and fun. I left me thinking and changing different problems. I would recommend this task next year."

"I was confused."

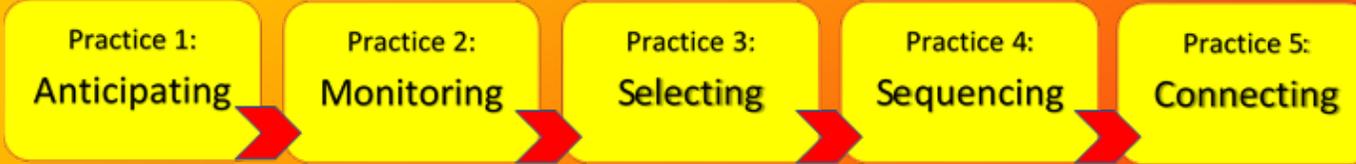
"The task helped me think logically."

Reflection

This discussion went really smoothly. I am usually good at facilitating a conversation, but planning the prompt and getting the students where I want to go is where I struggle. The framework from the book gave me guidance to slow down and plan effectively, which made the lesson much more successful.

FIVE PRACTICES FOR ORCHESTRATING PRODUCTIVE MATHEMATIC DISCUSSIONS - PART 2

Joan Masek
Alexander Hamilton High School



| Expected Changes in Practice | | | |
|------------------------------|--|--|--|
| ASPECT | CHANGE IN ROUTINE | EVIDENCE | ENSURE CONTINUATION |
| Planning | Task selection – high cognitive demand | Consult recommended resources – MPS instructional guide; Illustrative Math; Dan Meyer; colleagues. | <ul style="list-style-type: none"> Algebra team collaboration PLP Goal – Use of Lesson Planning Template |
| Teaching | Questioning Techniques – Monitoring Chart to implement use of Advancing Questions. | <ol style="list-style-type: none"> Monitoring Chart Formative Assessment | <p>QUESTION STEMS 5 Practices NCTM Professional Standards</p> |
| Reflection | Reflect on success of whole class discussion Select – Sequence – Connect | Student work samples Done / Not Done | <ul style="list-style-type: none"> Algebra team collaboration Use of Lesson Planning Template |

| Anticipating Student Strategies | | | |
|---------------------------------|---|---|-------|
| STRATEGY | ASSESSING QUESTIONS | ADVANCING QUESTIONS | ORDER |
| Guess and Check | Can you tell me what you did? How did you select your common multiplier? How did you select the next values? | Can you organize your data in a table or graph? So, you are saying.... Is there another way.... | 2 |
| Table | Can you tell me what you did? How did you select your common multiplier? How did you select the next values? | How are the values changing in the table? Is there another way to look at your data? So, you are saying.... Can you see a pattern? Since the ratios are different, how will you pick one? | 1 |
| Graph - DESMOS | Can you tell me what you did? How did you select your starting values? How did you get...? Why did you...? | Is there another way to look at your data? So, you are saying.... Can you see a pattern? What values are you changing to model the data? | 3 |
| Equation | Can you tell me what you did? How did you select your starting values? How did you get...? | So, you are saying.... Did you "check" this against your data? Can you predict other values? Is there another way to look at your data? | 4 |
| OTHER | | | |

Sequencing / Connecting

The sequencing of student strategies was:

- Table
- Guess and Check – Judgement
- DESMOS graph
- Equation – fit to data (DESMOS)

Selecting - Analysis of Artifacts

Tasks...

Monitoring Student Responses

Students given ONE STEP Investigation:

- How to organize the data?
- How to select multiplier?
- Creating the equation...
- Does the equation "fit" the data?

Teacher moves:

Teachers circulated in the room while students worked with their small groups.
Teachers are actively questioning students.

The Lesson Plan

THE TASK: INVESTIGATION RADIOACTIVE DECAY (ONE STEP) FITTING EXPONENTIAL MODELS TO DATA

The selection of a high-level cognitively demanding task is essential. This task promotes:

- Thinking
- Reasoning
- Problem solving

It has a low threshold and high ceiling. It is open-ended and allows multiple solution paths.

Students work in small groups. Use Think – Pair – Share

