

RESEARCH GROUP SPECIAL SESSIONS ABSTRACTS

ALGEBRA

"Recent Results in Quantum Rigidity"

Dr. Jason Gaddis (Wake Forest University)

A difficult problem in algebra is to determine the automorphism group of a particular ring. For example, the full automorphism group of the polynomial ring in three variables is not yet known. On the other hand, quantum rigidity says that automorphism groups of quantum algebras should be small in some sense. In this talk, I will present several examples of this phenomenon and discuss a recent strategy for computing automorphism groups by making use of the discriminant.

"An Interdisciplinary Case Study: Which Came First, the Mutation or the Antibiotic?"

Dr. Hema Gopalakrishnan (Sacred Heart University)

The mathematics and biology faculty at Sacred Heart University have developed interdisciplinary case studies for implementation in lower division undergraduate courses. Some of these case studies are directly linked to laboratory activities in specific biology courses. In this talk, I will present a case study based on the Luria-Delbrück fluctuation test that uses concepts from elementary statistics to understand which of two theories is true about bacterial mutations. I will also describe how it has been implemented in elementary statistics and biology classes.

"A proof of the peak polynomial positivity conjecture"

Dr. Pamela Harris (Williams College)

We say that a permutation $\pi = \pi_1 \pi_2 \cdots \pi_n$ on S_n has a peak at index i if $\pi_{i-1} < \pi_i > \pi_{i+1}$. Let $P(\pi)$ denote the set of indices where π has a peak. Given a set S of positive integers, we define $P(S; n) = \{\pi \in S_n : P(\pi) = S\}$. In 2013 Billey, Burdzy, and Sagan showed that for subsets of positive integers S and sufficiently large n , $|P(S; n)| = p_S(n) 2^{n-|S|-1}$ where $p_S(x)$ is a polynomial depending on S . They gave a recursive formula for $p_S(x)$ involving an alternating sum, and they conjectured that the coefficients of $p_S(x)$ expanded in a binomial coefficient basis centered at $\max(S)$ are all nonnegative. In this talk, we introduce a new recursive formula for $|P(S; n)|$ without alternating sums and we use this recursion to prove that their conjecture is true. This is joint work with Alexander Diaz-Lopez, Erik Insko, and Mohamed Omar.

"Arrowgrams: Tips and Pointers"

Dr. Kenneth Price (UW-Oshkosh)

The speaker invented arrowgrams as a way to explain some aspects of graded ring theory to his students. This includes a method to induce gradings on matrix rings, which was developed by S. Dascalescu, B. Ion, C. Nastasescu, and J. Rios Montes and expanded on by A. V. Kelarev, as well as an analogous method to induce gradings on incidence rings introduced by M. Jones.

An arrowgram is a secret message puzzle built on vertices connected by arrows, that is, a puzzle built on a directed graph. Some of the arrows are labeled by elements of a group. We call the value of an arrow its grade in order to be consistent with terminology used in graded ring theory. The solver uses a rule based on transitivity to determine the grade of every arrow and find out the secret message.

The grade of an arrow is an element of a group, which is called the grading group. An arrowgram with grading group Z_{20} , appeared in a 2011 issue of MAA Focus. This talk will give an account of arrowgram constructions that depend on the choice of the underlying group. Methods from combinatorics and linear algebra will be used.

"On Differential Subresultants and the Factorization of Stationary Schrödinger Operators"

Dr. Sonia L. Rueda (Polytechnical University of Madrid)

In 1928, J.L. Burchall and T.W. Chaundy established a correspondence between commuting differential operators and algebraic curves. With the discovery of solitons and the integrability of the KdV equation, by Gardner, Greene, Kruskal and Miura using the inverse spectral methods, their theory found applications to the study of partial differential equations called integrable (or with solitonic type solutions: Sine-Gordon, nonlinear Schrödinger, etc). Burchall and Chaundy had discovered the spectral curve (defined by the so called Burchall and Chaundy polynomial), which was later computed by E. Previato (1991), using differential resultants. The spectral curve allows an algebraic approach to handling the inverse spectral problem for the finite-gap operators, with the spectral data being encoded in the spectral curve and an associated line bundle (Krichever 1977).

In this work, we explore the benefits of using differential resultants to compute Burchall and Chaundy polynomials. We review the definition of the differential resultant of two ordinary differential operators and its main properties. We revisit Enma Previato's result about the computation of the spectral curve of two commuting differential operators using differential resultants. We use these results to establish the appropriate fields where commuting operators have a common factor, which can be computed using differential subresultants.

These results will allow us to give new explanations to some well known results related with the celebrated KdV hierarchy. We study some important families of pairs of commuting differential operators of rank one (Gesztesy and Holden), in particular, we study the centralizer of the a Schrödinger operator $L(u) = \partial^2 - u$ with u a differential indeterminate (we consider the Schrödinger operator in the stationary case, where $u = u(x)$ and $\partial = \partial/\partial x$). Using the differential resultant we describe recursive formulas to compute the defining polynomials of spectral curves γ_n associated to $L(u_s)$ with potential u_s satisfying the KdV $_n$, $n \geq s$ equation.

We present an algorithm to factor Schrödinger operators $L(u_s)$ with u_s satisfying the KdVs equation. Previous results of Brez and Gesztesy and Holden, for hyperelliptic curves, construct factorizations as formulas using γ -functions. As far as we know, there are no algorithms to obtain these factors. There are some differential recursive expressions but no final closed formulas are exhibited. The method we are presenting is effective and it points out the fact that closed formulas for factors of the Schrödinger operator over the curve can be obtained using: a global parametrization of the spectral curve (if it exists), and the subresultant formula operator. A key point to have an effective algorithm is to obtain a global parametrization of the curve, and how complicated the expression is depends on the parametrization.

These results, apart from allowing a direct computation show that, in order to build the Picard-Vessiot extension $PV(u_s)$ for $L(u_s)$, one can extend the coefficient field to the coefficient field of the curve and then to the Liouvillian extension given by $\gamma_x = \gamma$, with γ satisfying the Riccati equation $\gamma_x + \gamma^2 = u_s$. In this manner, we will describe the Picard-Vessiot fields $PV(u_s)$.

"Teaching Mathematics as a 'Learning Subject'"

Dr. Shubhangi Stalder (UW-Waukesha)

This presentation will describe the organization, content, and use of a new e-text that combines Developmental and Intermediate Algebra. I am also currently developing a similar e-text for College Algebra. Traditionally, the material taught in these lower-level math courses is organized in a linear fashion, and the problems tend to reinforce rote, procedural learning, thereby promoting something called a fixed mindset. My texts rearrange and streamline this material, and the newly designed problems promote a growth mindset. These changes promote mathematics as a "learning subject" with room for mistakes and growth instead of a "performance subject" (termed by Jo Boaler, Stanford professor of Math Education). I will share sample materials that help students to observe patterns and to become more comfortable in correctly communicating mathematics.

RESEARCH GROUP SPECIAL SESSIONS ABSTRACTS

ANALYSIS, APPLIED MATH, & COMPUTATIONAL MATH

“High Order Schemes for a Class of Cavity Flow Problems”

Dr. Saeed Dubas (University of Pittsburgh at Titusville)

Numerical Schemes of high order accuracy are developed for solving a class of incompressible, steady state Navier Stokes equations for cavity flow problems. The methods are efficient and reliable to obtain solutions over a range of Reynold's numbers which are in good agreement with other studies.

“Group Testing: From Syphilis to Sparse Fourier Transforms”

Dr. Mark Iwen (Michigan State University)

Periodic functions with a relatively small number of energetic Fourier coefficients appear in many applications including communication protocols, image processing problems, and numerical methods for solving some partial differential equations. In this talk we will discuss some algorithms for recovering such functions more quickly than possible via traditional discrete Fourier transform methods. In the process we will encounter world war two history, number theory, combinatorics, error correcting codes, and movie stars.

“Modeling a Diving Board”

Dr. Michael Karls (Ball State University)

The beam equation is a classic partial differential equation that one may encounter in an introductory course on boundary value problems or mathematical physics, which can be used to describe the vertical displacement of a vibrating beam. A diving board can be thought of as a cantilever beam, which is a bar with one end fixed and the other free to move. Using a video camera and physics demonstration software to record displacement data from a vibrating cantilever beam, we verify a modified version of the beam equation that incorporates damping and a forcing term.

“On Fractional Fourier Series and Integrals”

Dr. Ahmed Zayed (DePaul University)

Fourier series and integrals play an important role in many branches of science and engineering. Recently, fractional Fourier series and integrals have been introduced in several interesting applications in optics and signal processing. In this talk we will give a quick review of these fractional series and integrals and then report on some new results.

“Model Uncertainty and Selection in Operational Risk Modeling”

Dr. Daoping Yu (University of Central Missouri)

Model uncertainty arising from different ways treating the operational loss data collection threshold is investigated. Asymptotic normality of Value-at-Risk (VaR) estimates is established using the Delta method and asymptotic normality of Maximum-Likelihood-Estimation parameter estimates.

Evaluating the probability of overestimation/underestimation of the true target VaR in exponential and Lomax models, the truncated modeling approach turns out to be theoretically sound, while the shifted and naive approaches are fundamentally flawed. Using industry data of the external fraud type of event in the retail banking business line across major commercial banks in China for case study, the truncated lognormal, Lomax and Chambernowne models are compared. They all pass visual inspection of Quantile-Quantile plots as well as model validation by the Kolmogorov-Smirnov test and the Anderson-Darling test. However, they produce quite different VaR estimates. In the model selection procedure, those models are compared using Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and Information Complexity (ICOMP).

“A Bayesian Spatial Analysis of Mumps Data in Bulgaria”

Dr. Maya Zhelyazkova (Sofia University)

Bayesian spatial methods have been widely applied in different scientific areas such as epidemiological studies, image processing, fMRI data analysis and many others.

We will apply Bayesian hierarchical model with conditionally autoregressive (CAR) prior to a collection of weekly mumps data from 2000-2008 in Bulgaria.

We will generate a disease mapping of the crude standardized incidence ratio(SIR) across all regional centers. Similar mappings will also be produced for the smoothed relative risk.

The combination of methods for estimates of the relative risk is a powerful tool to identify high risk regions and may be used to inform local policies and programs.

RESEARCH GROUP SPECIAL SESSIONS ABSTRACTS

STATISTICS, PROBABILITY, & ACTUARIAL SCIENCE

“An Extended Euler-Maruyama Method for a Class of Stochastic Differential Equations”

Dr. Ram Adhikari (Rogers State University)

In this work we introduce a new class of weak numerical schemes (We call it Extended Euler-Maruyama scheme) for solving systems of Itô stochastic differential equations (SDEs). Weak order of convergence one is established under the suitable conditions. We also discuss about the numerical performance of our method with some examples. The proposed weak Extended Euler-Maruyama scheme has the potential to overcome some of the numerical instabilities that are often experienced when using explicit Euler method.

“Nonparametric Bayesian Density Estimation on Riemannian Manifolds”

Dr. Justin Jacobs (Sandia National Laboratories)

When continuous data are observed on a Riemannian manifold, the ability to estimate a density is contingent on the properties of the manifold. Instead of performing density estimation using extrinsic methods, we propose an intrinsic method for estimating a density on a manifold of interest. Assuming the density function is contained in the class of square integral functions on the manifold we defer to the spectral properties under the Laplace-Beltrami equation defined using a particular local coordinate chart. An example on well-known spaces will be investigated.

“Comparing the Riskiness of Dependent Portfolios”

Dr. Ranadeera Samanthi (Central Michigan University)

A nonparametric test based on nested L-statistics to compare the riskiness of portfolios was introduced by Brazauskas, Jones, Puri, and Zitikis (2007). In this work, we investigate how the performance of the test changes when insurance portfolios are dependent. To achieve that goal, we perform a simulation study using spectral risk measures. Further, three insurance portfolios are generated, and their interdependence is modeled with the three-dimensional elliptical copulas. It is found that the presence of comonotonicity makes the test liberal for all the risk measures under consideration. We illustrate how to incorporate such findings into sensitivity analysis of decisions.

“Unbiased Generalized Linear Models”

Dr. John Wood (HumanaOne)

We use the quasi-likelihood methodology to fit generalized linear models so that the model prediction satisfies certain moment conditions for arbitrary mean, link, variance or quasi-likelihood functions. We unify the quasi-likelihood method and method of moments. When restricted to the classical exponential family of univariate distributions and their likelihood functions, the link functions that we derive are the classical canonical link functions. We apply a log-linear or factor model to simulated loss data and a truncated exponential mean model to simulated right censored survival data.

RESEARCH GROUP SPECIAL SESSIONS ABSTRACTS

TOPOLOGY & DYNAMICS

“Word Calculus in the Fundamental Group of the Menger Cube”

Dr. Hanspeter Fischer (Ball State University)

The fundamental group of the Menger cube is uncountable and not free, although all of its finitely generated subgroups are free. It contains an isomorphic copy of the fundamental group of every one-dimensional separable metric space and an isomorphic copy of the fundamental group of every planar Peano continuum.

We give an explicit and systematic combinatorial description of the fundamental group of the Menger cube and its generalized Cayley graph in terms of word sequences. The word calculus, which requires only two letters and their inverses, is based on Pasyukov’s partial topological product representation and can be expressed in terms of a variation on the classical puzzle known as the Towers of Hanoi. This is joint research with Andreas Zastrow (University of Gdańsk, Poland).

Title: “From Recreation to Research”

Dr. Paul Fonstad (Franklin College)

How do you keep research going when you work at a school where the focus for faculty is on teaching and service? By letting your service inspire you! This talk will examine how I gained research ideas by running a weekly math problem competition and by going to a swim meet, and will discuss the results of the inspired research.

“Can We Use Minimal Sets to Describe Omega Limit Sets of Maps of the Interval”

Dr. Michael W. Hero (University of Iowa)

This talk will introduce two open questions in this setting. One question regards the decomposition of omega limit sets into minimal sets and the other regards a very special approximation of the logistic map by a family of mapping of finite sets. The talk will be very accessible to students.

“The Geometry of Domino Tilings”

Dr. David Radcliffe (Co-Founder & Lead Software Developer at GogyUp)

There are many ways to cover a chessboard with dominoes; 12,988,816 to be precise. But they are all connected by a simple operation, called a flip. The set of all domino tilings of a rectangle is thus a connected graph — two tilings are adjacent if and only if they differ by a flip. We will explore the structure of this graph, and we will see how determinants can be used to enumerate domino tilings.

“The World’s Most Complicated Proof that the Complete Graph on 5 vertices is Non-Planar”

Dr. Timothy Schroeder (Murray State University)

This talk will explain how Coxeter groups and corresponding H^2 -homology theory can be used to prove that K_5 , the complete graph on 5 vertices, is non-planar. The method works for some other graphs, too. Moreover, we will describe how H^2 -technology can be used to estimate the genus of a graph.