

231 Gateway 3 Practice Test - Graphs

No uses of Calculators; No Partial Credit. 30 minutes to finish test. More space will be provided on the actual test.

1. Consider the function $f(x) = 3x^4 + 2x^3 - 3x^2 + 2$
- (a) (10 pts) Find all of the critical numbers of f .
 - (b) (10 pts) Give the points $(x, f(x))$ of the absolute minimum and absolute maximum for f on the interval $[0, 1]$.

2. (10 pts) Find the critical numbers where f has a local maximum or minimum on the given domain and identify each point as a local maximum or local minimum. If there is no local maximum and no local minimum, explain (briefly) why.

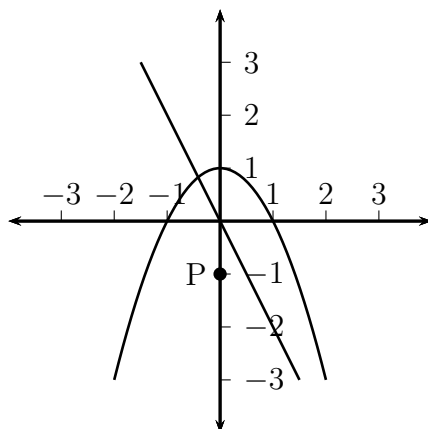
$$f(x) = x^2 + \frac{3}{x}, 0 < x < \infty.$$

3. For the given derivative of a function f , $f'(x) = (x + 1)(x + 2)$,

- (a) (5 pts) What are the critical numbers of f ?
- (b) (5 pts) On what intervals is f increasing?
- (c) (5 pts) On what intervals is f decreasing?
- (d) (5 pts) At what critical numbers, if any, does f assume a local maximum?
- (e) (5 pts) At what critical numbers, if any, does f assume a local minimum?

4. The graphs of the first and second derivative of a function $y = f(x)$ are shown.

- (a) (3 pts) On what intervals is f increasing?
- (b) (3 pts) On what intervals is f decreasing?
- (c) (3 pts) On what intervals is f concave up?
- (d) (3 pts) On what intervals is f concave down?
- (e) (8 pts) Add to the picture a sketch of the approximate graph of f , given that the graph passes through the point P.



5. (2 pts each) Let $f(x)$ be a function such that f'' is continuous. State whether f has a Local Maximum, a Local Minimum, Neither, or if the data is inconclusive at each of the following values of x .

x	$f'(x)$	$f''(x)$	
1	0	8	
2	9	6	
3	0	-5	
4	7	0	
5	0	0	

6. Use the following information to answer this problem:

$$\lim_{x \rightarrow -1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -1$$

$$\lim_{x \rightarrow \infty} f(x) = 1$$

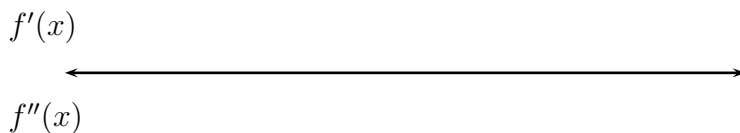
$$f'(x) < 0 \text{ for } x \in (-\infty, -1) \cup (-1, 2)$$

$$f'(x) > 0 \text{ for } x \in (2, \infty)$$

$$f''(x) < 0 \text{ for } x \in (-\infty, -1) \cup (3, \infty)$$

$$f''(x) > 0 \text{ for } x \in (-1, 3)$$

(a) (6 pts) Label all of the important x -values derived from the information provided above on the number line below. Then indicate the intervals along this line where $f'(x)$ and $f''(x)$ are positive or negative.



(b) (9 pts) Sketch the graph of a function that satisfies all of the given conditions. No formulas are required but be sure to label all of the important values on the coordinate axes.