Ph.D. Qualifying Exam

September 15, 2017
1:00-5:00pm

Electrical Engineering

Part II

Instructions:

This is a closed-book/closed-notes exam module, four hours in duration. Please answer all problems in two areas you have previously selected.

Write your name and student ID below.

Full Name: ____________________________________________

Student ID: ____________________________________________

Please write the last four digits of your student ID number on EVERY PAGE, but DO NOT write your name on any other pages of the exam.

Blank paper is provided if you need additional space to show your work. PLEASE INDICATE THE PROBLEM NUMBER ON EACH ADDITIONAL PAGE YOU USE.

Good luck!
Final grade for this section: _________ out of 40 points

Signal Processing
Question 1, ElecEng 410

(a) Given the Discrete Fourier Transform (DFT),

\[ X(k) = \sum_{n=0}^{N-1} x(n) \ e^{-j2\pi kn/N} = \sum_{n=0}^{N-1} x(n) \ W_N^{kn} \]

where

\[ W_N = e^{-j2\pi/N} \]

Derive the Fast Fourier Transform (FFT) decimation in frequency.

(b) How many calculations for the DFT in terms of N ?
(c) How many calculations for the FFT in terms of N ?
Question 2, ElecEng 410

(a) For an N-tap FIR filter sketch the direct form realization. How many multipliers per sample are required? (as an example you may choose N=8)

(b) For an FFT calculation of an N-tap FIR filter sketch the realization. How many multipliers per sample are required? (as an example you may choose N=8 and FFT size 256)

(c) Which realization is more computationally efficient for different size N?
Question 3, ElecEng 420

Problem 3 (25 points)

Let $X$ be a random variable with density function

$$p_X(x) = \begin{cases} 
A(1 - |x|), & x \in [1, +1] \\
0, & \text{otherwise}
\end{cases}$$

(1)

where $A > 0$ and

Let random variable $Y$ be related to $X$ by

$$Y = g(X) = \begin{cases} 
-X, & x < 0 \\
2X, & x \geq 0
\end{cases}$$

(2)

a.) Find the value of $A$ and sketch $p_X(x)$

b.) Find $E[Y]$.

c.) Find the PDF of $Y$, $P_Y(y)$.

d.) Find and sketch the pdf of $Y$, $p_Y(y)$. 
Question 4, ElecEng 420

Problem 4 (25 points)

In Fig. 1, $X(t)$ is a zero-mean wide-sense stationary random process. Furthermore, $\Theta_1$ and $\Theta_2$ are random variables that are uniformly distributed in $[0, 2\pi]$ and are independent of each other and $X(t)$.

a.) Find the correlation function of $Y(t)$ in terms of the correlation function of $X(t)$.

b.) Find and sketch the power density spectral (PDS) of $Y(t)$. Note for any $s(t)$ with Fourier transform $S(\omega)$, the Fourier transform of $s(t) \cos \omega_0 t$ is $[S(\omega - \omega_0) + S(\omega + \omega_0)]/2$.

c.) Find the sketch the PDS’s of $Z(t)$ and $U(t)$.

d.) Find and sketch the PDS of $V(t)$. What does the system in Fig. 1 do?
Final grade for this section: __________ out of 40 points

Controls
• Attach extra sheets as needed to provide space for your answers.

• Clearly identify each of your answers with the corresponding problem number and part, such as 1.A, 2.B.ii, etc.

• Show your work, to show how you arrived at each answer.

• Several questions marked (Essay Question) are included. Answer these with one or more complete sentences and complete and logical thoughts. Responses with incomplete sentences and incompletely described thoughts will be heavily discounted.

Some background equations

• Controls measures from pole locations, given a pole at

\[ s = \alpha + j\omega = \omega_n \angle \theta_p \]  

Then percent overshoot is approximated by

\[ P.O. = 100 \times e^{-\frac{\pi \omega}{\omega_n}} \]  

Rise time and settling time are approximated by

\[ t_r \approx \frac{1.8}{\omega_n}, \quad t_s \approx \frac{4}{\alpha} \]  

• Closed-loop bandwidth can be estimated from open-loop cross-over frequency

\[ \omega_b \approx \sqrt{2} \omega_c \]  

• Convert from continuous to discrete equivalent poles

\[ z_i = e^{s_i T_s} \]  

where \( s_i \) is a continuous-time pole, \( T_s \) is the sample period, and \( z_i \) is the discrete time equivalent pole.
Question 1, ElecEng 474, 574

• Write a 1 page essay discussing similarities and differences continuous-time and discrete-time control system analysis and design. The next page provides additional space. Address at least the items below. It should be clear where in your essay each items is addressed. Use specific and detailed examples.

– When would one continuous-time or discrete-time methods be applied? Give an example of each.

– How is analysis similar between the two?

– How is analysis different between the two?

– How is control design similar between the two?

– How is control design different between the two?
Question 2, ElecEng 474, 574

A) For the system of figure 1, determine an expression for the closed loop transfer function

\[ T_{ry}(s) = \frac{Y(s)}{R(s)} \]

B) For the system of figure 1, determine an expression for the closed loop transfer function

\[ T_{ve}(s) = \frac{E(s)}{V_s(s)} \]

C) (Essay Question) When transfer function \( T_{ry}(s) \) is evaluated, how are inputs such as \( d(t) \) considered?

D) What is the transfer function from \( d(t) \) to \( V_s(t) \), that is \( \frac{D(s)}{V_s(s)} \)?
Question 3, ElecEng 474, 574

- The bode plot for a plant, \( G_{p3}(s) \), is seen in figure 3. Considering this bode plot, answer the questions below.

A) With the system in closed loop with proportional control, as seen in figure 2, what is the range of \( K_p \) for which the system will be closed-loop stable.

![Block diagram of \( G_{p3}(s) \) with proportional control \( K_p \).](image)

B) What value of \( K_p \) will give a phase margin of 45 degrees?

C) For the value of \( K_p \) you determined in question (B), what will be the gain margin?

D) For the value of \( K_p \) you determined in question (B), what will be the level of tracking accuracy at \( \omega_t = 0.1 \, [\text{rad/sec}] \)?

E) (True / False, circle one) Bode-plot based methods can only be applied to continuous-time systems, they are not applicable to discrete-time systems.
Figure 3: Bode plot of a plant, $G_p(s)$. 
Question 4, ElecEng 474, 574

- For the system illustrated in figure 4, answer the following questions.

\( T_s = 0.1 \) seconds

\[
\begin{array}{c}
\text{Controller} \\
\frac{z - 0.6}{z - 0.9} \quad \frac{z}{z - 0.9} \\
\text{Plant}
\end{array}
\]

Figure 4: Block diagram for a discrete system.

A) Is the closed-loop system stable with \( K_p = 1.0 \)?

B) What is the system type (type 0, type I, etc., or do discrete systems not have a type)?

C) On the back of the previous page, sketch the open-loop pole-zero diagram, including the stability boundary.

D) On the back of the previous page, sketch the root locus for this system. Write a few sentences discussing system characteristics as \( K_p \) is increased from zero to a large value.
Final grade for this section: \[ \text{___________} \] out of 40 points

Nanoengineering
Question 1, ElecEng 482, 588

An electron propagates freely with momentum $\vec{p} = p_0 \hat{x}$. Using momentum operator, $\hat{p}_x$

a) write down the wavefunction of the electron and obtain kinetic energy operator, $\hat{T}_k$

b) calculate the average momentum, $<p_x>$ and kinetic energy of the electron $<T_k>$

c) check if $\hat{p}_x$ and $\hat{T}_k$ commute. Explain your findings. (Recall that commutator of $a$ and $b$ is defined as $[a,b] = ab - ba$)
Question 2, ElecEng 482, 588

Derive E(k) formula for quantum wire made out of semiconductor with the band gap of $E_0$ and has rectangular crosssection $a \times b$ where $b=3a$. Plot E(k) for 2 lowest energy states for electron and holes. For calculations assume that effective masses of electron and holes are the same.
Question 3, ElecEng 482, 588

Write down Fermi-Dirac distribution function, \( f(E) \) and

a) explain its meaning
b) investigate its behavior at two different temperatures: 0 K and room temperature for a metal with \( E_F \) of ~1 eV.
c) explain the role of free electrons behind the electrical conductivity as a function of their energy at the room temperature.
Find eigenfunctions of stationary Schrodinger equation for infinite potential well of width $b$, i.e. $U(x) = \begin{cases} 0, & 0 < x < b \\ \infty, & x < 0, x > b \end{cases}$. Normalize the wavefunctions and find $\langle x \rangle$, $\langle x^2 \rangle$ and $\langle (\Delta x)^2 \rangle$. 
Final grade for this section: __________ out of 40 points

Power
Question 1, ElecEng 471, 575

A three-phase PWM inverter drive is used to provide 450V, 60Hz voltage to an induction motor under nominal rated conditions with the motor having the following parameters:

\[ R_s = 0.05 \ \Omega, \ R_r = 0.1 \ \Omega, \ X_{ls} = X_{lr} = 1 \ \Omega \]

The motor rated speed is 1760 rpm and rated stator current is 72 A. A constant V/Hz control is used to maintain magnetizing flux at

\[ \lambda_m = 0.87 \ \text{Webers} \]

The inverter drive DC link is at 650V and it operates at constant switching frequency with a switching frequency index of \( m_f = 200 \). (Note that number of poles and slip may be required to solve the problems below correctly—these can be found from the provided information).

(a.) What is the switching frequency of the drive?

(b.) Using the table below, find the modulation index under nominal rated conditions and the three highest switching frequency harmonic currents. \textit{Assume that under this condition the rotor current amplitude is 99\% of the stator current amplitude and that at switching frequency and its harmonics the magnetizing reactance is nearly infinite.}
(c.) If the drive is operated at 50% of rated speed, find the fundamental current and the three highest switching frequency harmonic currents. Assume that with constant V/Hz control the slip is also relatively constant (true for fan or pump load) and that under this condition the rotor current amplitude is 99% of the stator current amplitude and that at switching frequency and its harmonics the magnetizing reactance is nearly infinite.

Table 8-1  Generalized Harmonics of $e_{amb}$ for a Large $m_r$

<table>
<thead>
<tr>
<th>$m_r$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.242</td>
<td>1.15</td>
<td>1.006</td>
<td>0.818</td>
<td>0.601</td>
</tr>
<tr>
<td>$m_r \pm 2$</td>
<td>0.016</td>
<td>0.061</td>
<td>0.131</td>
<td>0.220</td>
<td>0.318</td>
</tr>
<tr>
<td>$m_r \pm 4$</td>
<td></td>
<td></td>
<td>0.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2m_r \pm 1$</td>
<td>0.190</td>
<td>0.326</td>
<td>0.370</td>
<td>0.314</td>
<td>0.181</td>
</tr>
<tr>
<td>$2m_r \pm 3$</td>
<td>0.024</td>
<td>0.071</td>
<td>0.139</td>
<td>0.212</td>
<td></td>
</tr>
<tr>
<td>$2m_r \pm 5$</td>
<td></td>
<td>0.013</td>
<td>0.033</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3m_r$</td>
<td>0.335</td>
<td>0.123</td>
<td>0.083</td>
<td>0.171</td>
<td>0.113</td>
</tr>
<tr>
<td>$3m_r \pm 2$</td>
<td>0.044</td>
<td>0.139</td>
<td>0.203</td>
<td>0.176</td>
<td>0.062</td>
</tr>
<tr>
<td>$3m_r \pm 4$</td>
<td>0.012</td>
<td>0.047</td>
<td>0.104</td>
<td>0.157</td>
<td></td>
</tr>
<tr>
<td>$3m_r \pm 6$</td>
<td></td>
<td>0.016</td>
<td>0.044</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4m_r \pm 1$</td>
<td>0.163</td>
<td>0.157</td>
<td>0.008</td>
<td>0.105</td>
<td>0.068</td>
</tr>
<tr>
<td>$4m_r \pm 3$</td>
<td>0.012</td>
<td>0.070</td>
<td>0.132</td>
<td>0.115</td>
<td>0.009</td>
</tr>
<tr>
<td>$4m_r \pm 5$</td>
<td></td>
<td>0.034</td>
<td>0.084</td>
<td>0.119</td>
<td></td>
</tr>
<tr>
<td>$4m_r \pm 7$</td>
<td></td>
<td>0.017</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: $(V_{amb})^2/V_d = (V_{amb})^2/V_d$ is tabulated as a function of $m_r$. 
Question 2, ElecEng 471, 575

Additional EE 572 Question

For the circuit below a synchronous generator is feeding a phase controlled rectifier operating as a DC motor drive. $L_u$ is very large and $L_u >> L_L$. The output voltage $v_o$ is adjusted using the firing angle $\alpha$ to control the motor speed (which is proportional to the back emf voltage $e_u$). Assume that the motor is loaded to a sufficient level of armature current to ensure that the phase controlled rectifier operates in continuous mode.

![Circuit Diagram]

a.) Draw the waveshapes for $i_A$, $i_b$, $i_C$ and $v_d$. Be sure to include the effect of $L_u$ on the commutating waveforms.

b.) Derive expressions for RMS line to neutral voltage, $v_{an}$, and average armature current, $i_w$, in terms of $v_d$, $e_u$, $\alpha$ and $R_u$. In this case it is okay to neglect the impact of $L_u$. 
Question 3, ElecEng 471, 575

A Buck-Boost converter is shown below.

(a) Applying the principles of volt-second balance, show the steps to derive the expression for the ratio of output voltage to input voltage as a function of duty cycle (D). *(Note that the output voltage is inverted from the input voltage)*

(b) If the Buck-Boost converter has an input of 10V and is to have an output of -20V supplying a 50W load. The switching frequency is 50kHz. Find the duty ratio and calculate the inductor inductance (L) such that the inductor current is no more than 10% of the average inductor current. Also, find the average load current (\(i_o\)) and input current (\(i_d\)).
Question 4, ElecEng 471, 575

(a) A single phase transformer is rated 25KVA and 1300V/200V. Please (a) configure this transformer into a step-up auto-transformer 200V/1500V and (b) calculate the input and output currents of this auto-transformer

(b) A generator supplies the load through a standard ∆-Y transformer. If the symmetrical component currents from the Y side of the transformer to the load are \(I_{a1} = -j1.5\), \(I_{a2} = -j1.5\), and \(I_{a0} = -j2.5\). Please find the line current \(I_C\) on the ∆ side.
Final grade for this section: __________ out of 40 points

Photonics
Question 1, ElecEng 464, 565

The following three questions relate to the following diagram for an interface between two media with refractive indices $n_1$ and $n_2$. You can draw your answer on the diagram.

![Diagram of interface between two media with refractive indices $n_1$ and $n_2$.]

a) Assume $n_1 > n_2$. Draw an incident ray with angle of $\theta_i$ and a transmitted ray with refraction angle of $\theta_t$.
b) Draw the reflected ray
c) The ray direction is the propagation direction. Draw the electric and magnetic field directions for incident, transmitted and reflected waves for a s-polarized incident wave.
d) If dispersion parameter $D_\lambda > 0$, how does delay depend on frequency? Label in the following diagram the relative temporal position of low and high frequencies.

e) An 1mW Gaussian beam with beam radius $W = 0.1$ mm. Find its peak intensity.

g) Estimate power transmission of a wave traversing a thin layer of GaAs with refractive index of 3.5 which has air as its outside medium. Assume normal incidence of the wave.
a. Consider a crystal with $n_e = 1.553$ and $n_o = 2.052$ at wavelength $\lambda_o = 1\mu\text{m}$.
   i. Draw the k surfaces for the ordinary wave and extraordinary wave.
   ii. For an interface between the crystal and a medium with refractive index $n_2 = 2$ as show in the diagram. Find the angle $\theta_{2o}$ between the normal and the ordinary wave and the angle $\theta_{2e}$ between the normal and the extraordinary wave in the medium.

![Diagram of optic axis and angle](image)

b. Design a optical modulator with a rotator and two polarizers. Be sure you specify the orientation of the polarizers and find the Jones matrix of the modulator. Show that the modulator can vary the intensity by changing the angle of polarization rotation.
Question 3, ElecEng 464

A 2-km-length multimode fiber has a modal dispersion of 1 ns/km and a chromatic dispersion of 100 ps/km• nm. If it is used with an LED of linewidth 40 nm, (a) what is the total dispersion? (b) Calculate the bandwidth (BW) of the fiber.
A fiber has attenuation of 0.6 db/km at 1300nm and 0.3 dB/km at 1550nm. Suppose two optical signals are launched into the fiber simultaneously; an optical power of 150 μW at 1300nm and an optical power of 100 μW at 1550nm. What are the power levels in μW of these two signals after 8km propagation in the fiber. (10)
Final grade for this section: __________ out of 40 points

Electronics
Question 1, ElecEng 451, 541, 562

The simplified layout of a CMOS complex logic circuit is given below. Draw the corresponding circuit diagram, and equivalent Boolean function realized by this circuit.
Question 2, ElecEng 451, 541, 562

Please design a two-input XOR gate through either Combinational logic gate circuits or transmission gates.
Question 3, ElecEng 451, 541, 562

For circuit shown below, please calculate the differential voltage gain if \((W/L)_{1,2} = 50/0.5\), and \((W/L)_{3,4} = 50/1\). Assume mobility ratio between NMOS and PMOS is 1.5.
Question 4, ElecEng 451, 541, 562

The circuit shown below is designed with $R_3=1\,\text{k}\Omega$ and a current of $50\,\mu\text{A}$ through it. Please calculate $R_1$, $R_2$ and $n$ for zero Temperature Coefficient.
Final grade for this section: __________ out of 40 points

Biomedical
Define the following -

a) Half-cell potential
b) Bundle of His
c) Node of Ranvier
d) Absolute refractory period
Question 2, ElecEng 437

Find the Fourier transform of each image of the right column in the left column

(a)

(b)

(c)

(d)
Question 3, ElecEng 437

Plot the A-mode ultrasound scan of a tissue plotted in the figure below. In the plot y axis should be the echo intensity in db and x-axis in microseconds. Ignore reflected signal at transducer/fat interface and assume the intensity of entering signal is 0 bd. At a transducer frequency of 5 MHZ, the linear attenuation coefficient for muscle is 5 db/cm and for fat is 7 db/cm. (10 points)

Table: Acoustic property of biological tissue:
Question 4, ElecEng 437

(10 pt) Consider a MRI session. Assume the static field strength is $B_0 = 1.5$ Tesla, the $z$-gradient is $Gz = 2$ gauss/mm, and the gyromagnetic ratio is $\gamma = 4.258$ KHz/gauss. We would like to image a slice with center position $z_c = 10$ cm, thickness $\Delta z = 1$ mm. Find the frequency range of the RF waveform required to excite this slice.
Student ID:____________
Student ID: ____________