MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part I
Mathematics and Fundamentals

Closed Book/Notes

April 10, 2008
Thursday
1:00 pm - 5:00 pm

The problems are:		Enter problem number(s) that you selected
Ordinary Differential Equations	2 problems	( ) ( )
Partial Differential Equations	2 problems	( ) ( )
Linear Algebra	2 problems	( ) ( )
Calculus	2 problems	( ) ( )
Numerical Methods	2 problems	( ) ( )

The student should select 8 problems out of 10 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your assigned number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

Ordinary Differential Equation #1

Find the solution of

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 4y = 0,$$

with initial conditions of y(0) =1 and

$$\frac{dy}{dt}(0) = -3$$

.

Ordinary Differential Equation #2

Find the solution of the equation

$$(1 + x^2)\,(dy - dx) = 2xydx$$

for which $y = 1$ when $x = 0$.

Partial Differential Equation #1

Consider the following partial differential equation:

$$\frac{\partial\theta}{\partial t}=\alpha\frac{\partial^2\theta}{\partial x^2}+\frac{u'''}{\rho c} \qquad 0<x<L,\ 0<t<\infty$$

Where α, and $\frac{u'''}{\rho c}$ are constants.

The initial and boundary conditions are:

$$\theta(x,0)=0$$
$$\frac{\partial\theta(0,x)}{\partial x}=0$$
$$\theta(L,t)=0$$

Find the solution θ (x,t).

Partial Differential Equation #2

The differential equation to describe the transient heat conduction is given as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

with the boundary conditions of T as:

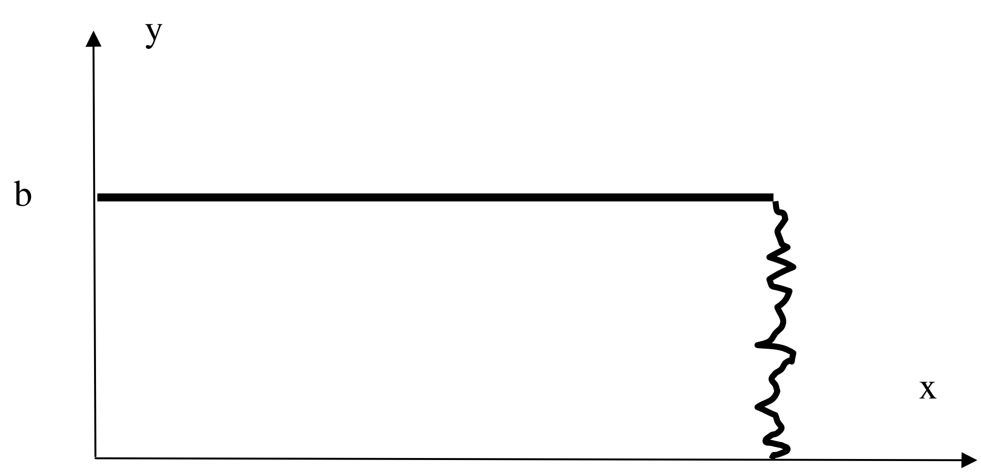
B.C.:

$T = 0 \qquad at \;\; x = 0$

$T = f(x) \qquad at \;\; y = 0$

$T = 0 \qquad at \;\; y = b$

Obtain an expression for the steady state temperature distribution T(x,y) in a semi-infinite strip $0 \le x \le \infty, \qquad 0 \le y \le b$.



Linear Algebra #1

Let A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

Find a real orthogonal matrix P for which $P^t AP$ is diagonal.

Show all of your work

Linear Algebra #2

Given the matrix

$$[A] = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Compute A^n for an arbitrary n.

Calculus #1

Using the method of Lagrange multipliers, maximize the function P defined by

$$P(x,y,z) = 1000\,z - 600\,x - 300\,y,$$

subject to the contraint: $z = x/3 + y/5 + 4 - 1/x - 2/y$.

Calculus #2

Calculate the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

Numerical Methods #1

Consider the following two-dimensional parabolic equation in cylindrical coordinates:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

Assuming: $\partial u/\partial r = 0$ at $r=0$ due to symmetry at $r=0$. It can be seen that the second term in the right hand side ends up with 0/0 as $r \to 0$. Please derive the corresponding equation at this point that can be solved without the singularity. Also, derive its explicit form of the finite difference solution at this point. Use h and k as spatial and temporal grid size, respectively.

Numerical Methods #2

Consider the following numerical discretized method that consists of two steps:

$$f^{n+1/2} = f^n - \frac{\Delta t}{2} \frac{u(f_{i+1}^{n+1/2} - f_{i-1}^{n+1/2})}{2\Delta x}$$

and

$$f^{n+1} = f^n - \Delta t \ \frac{u(f_{i+1}^{n+1/2} - f_{i-1}^{n+1/2})}{2\Delta x}$$

where superscripts denote the time step and subscripts denote the space step. Also Δt and Δx represent the time and space increment, respectively.
u=velocity=constant

Analyze for stability using the von Neumann method.

MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Thermal Science Stem

Open Book/Notes

April 11, 2008
Friday
1:00 pm - 5:00 pm

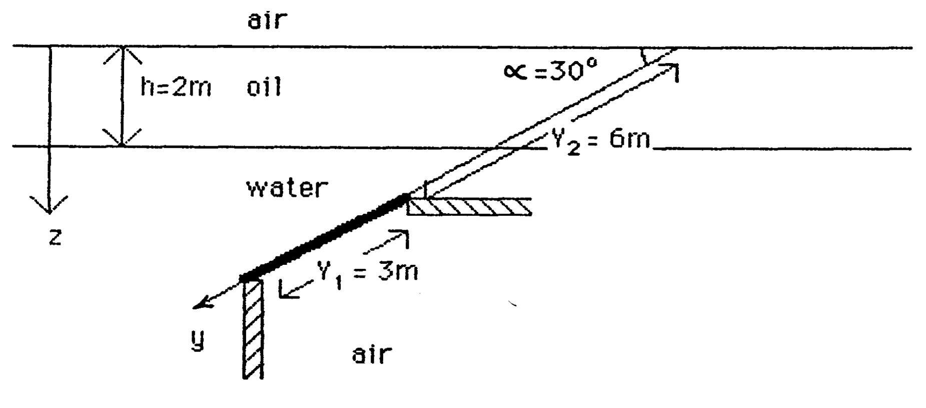
The problems are:		Enter problem number(s) that you selected
Fluid Mechanics	3 problems	( ) ( ) ( )
Thermodynamics	3 problems	( ) ( ) ( )
Heat Transfer	3 problems	( ) ( ) ( )

The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in (   ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your assigned number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

Fluid Mechanics #1

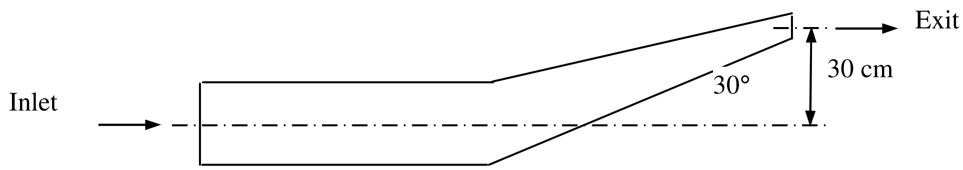
A 2m layer of oil with γ_0 = 7800 N/m^3 is floating on top of water with γ_w = 9810 N/m^3. A flat plate of length Y_1 = 3 m is submerged in the water as shown below. The plate has a depth, d, of 5 m into the page.

a) What is the (gage) pressure p(z) at any point in the water? Note that z is the vertical distance into the fluid. You can leave your answer in terms of γ's and h.
b) What is the net force acting on the plate? (Calculate a number.)



Fluid Mechanics #2

A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upwards 30° while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the exit. The elevation difference between the center of the exit and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gage pressure at the center of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place.

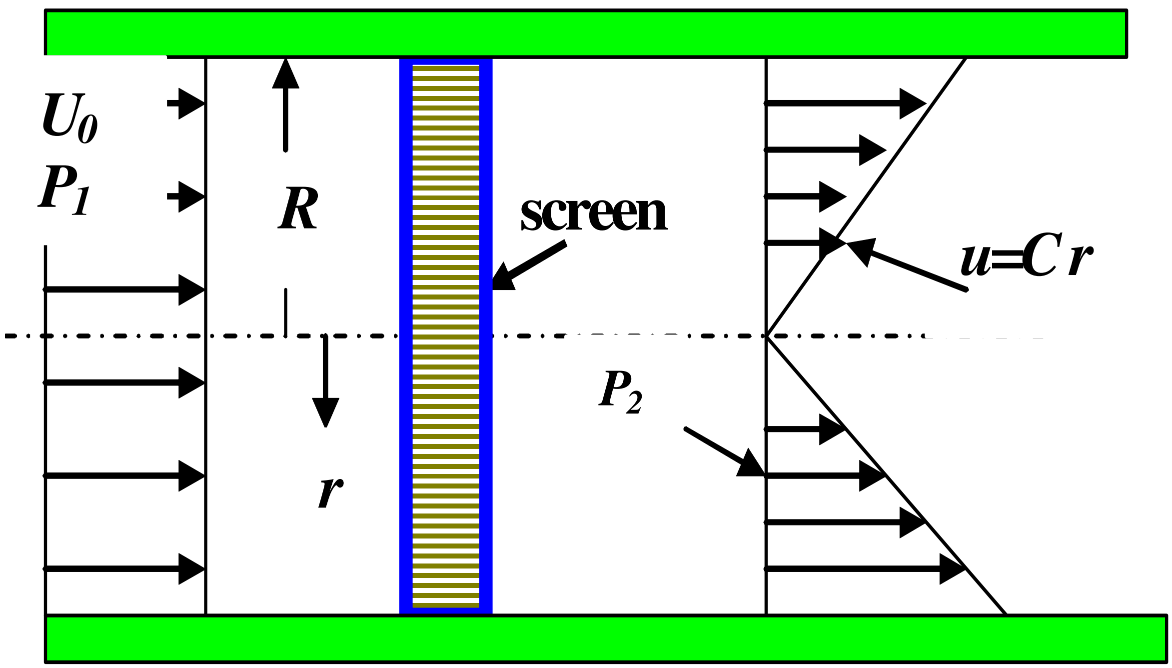


Fluid Mechanics #3

A uniform velocity profile air flow enters the circular pipe (with a radius R) at the velocity U_0. A variable honeycomb screen produces a linear and axisymmetric velocity profile, $u = C\,r$, as indicated in the figure shown below. The static-pressures upstream and downstream of the honeycomb screen are P_1 (at the entrance) and P_2 (at the section where the velocity profile becomes triangular shape), respectively. The pressure is assumed to be constant in the radial direction.

(1) Determine the velocity coefficient C.

(2) Obtain the drag force due to the screen and the friction in terms of P_1, P_2, ρ, R, and U_0.



Thermodynamics #1

Two kg of saturated water vapor at 1 MPa undergoes a reversible, isothermal expansion until the pressure reaches 0.15 MPa. Determine the heat transfer and the work done for this process. The system boundary temperature is the same as the process temperature.

Thermodynamics #2

A piece of aluminum (c_p=0.903 kJ/kg-K) with a mass of 50 kg is initially heated to 300^{o}C. The aluminum is then placed into an empty refrigerator, and is cooled until the temperature is 10^{o}C. The refrigerator operates on a vapor-compression refrigeration cycle. R-134a enters the evaporator of the cycle at –8^{o}C and a quality of 0.20, and absorbs heat at constant pressure before exiting the evaporator as a saturated vapor. The volumetric flow rate of the R-134a leaving the evaporator is 0.005 m^3/s. The coefficient of performance of the refrigerator is 2.50. Determine,

(a) the time required to cool the aluminum, and
(b) the total amount of work used by the compressor in the cycle, during the cooling process.

Thermodynamics #3

The centrifugal air compressor of a gas turbine receives air from the ambient atmosphere where the pressure is 1 bar and the temperature is 300 K. At the discharge of the compressor the pressure is 4 bar, the temperature is 500 K, and the velocity is 100 m/s. The mass flow rate into the compressor is 15 kg/s. Determine the power required to drive this compressor using (a) constant specific heat and (b) air tables. Show all work.

Heat Transfer #1

Consider a 5mx5mx5m cubical furnace whose surfaces closely approximate black surfaces. The base, top, and the side surfaces of the furnace are maintained at uniform temperatures of 800, 1500, and 500 K, respectively. Determine (a) the net rate of radiation heat transfer between the base and the side surfaces, (b) the net rate of radiation heat transfer between the base and the top surface, and (c) the net total radiation heat transfer from the base surface.

Heat Transfer #2

A 2 kg household iron is made of aluminum and has a 500 W heating element. The ambient temperature is 20^{o}C, and the surface area of the iron is 0.05 m^2, while the heat transfer coefficient may be assumed constant at 10 W/m^{2}K. How long will it take the iron to reach 100^{o}C after being turned on? $C_{aluminum}$ = 900 J/kgK.

Hint: use lumped system, and start from first law of thermodynamics.

Heat Transfer #3

A cross-flow heat exchanger with both fluids unmixed is to heat water with hot exhaust gas. The water enters the tubes at 25°C at a rate of 1 kg/s, while the exhaust gas enters the exchanger at 200°C at a rate of 2 kg/s. The total heat transfer surface area is A = 30 m^2 and the overall heat transfer coefficient is U = 120 $W/(m^2 \cdot °C)$. The specific heats for the exhaust gas and water may be taken as $C_{p,h}$ = 1100 J/(kg·°C) and $C_{p,c}$ = 4181 J/(kg·°C), respectively. Calculate (i) the total heat transfer rate, (ii) the outlet exhaust gas temperature, and (iii) the outlet water temperature. Show all work.

MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Machine Design Stem

Open Book/Notes

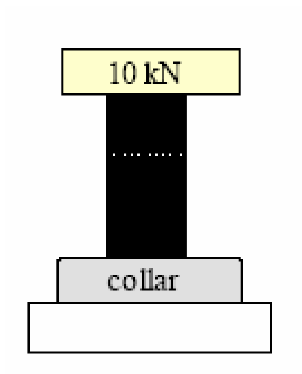
April 11, 2008
Friday
1:00 pm - 5:00 pm

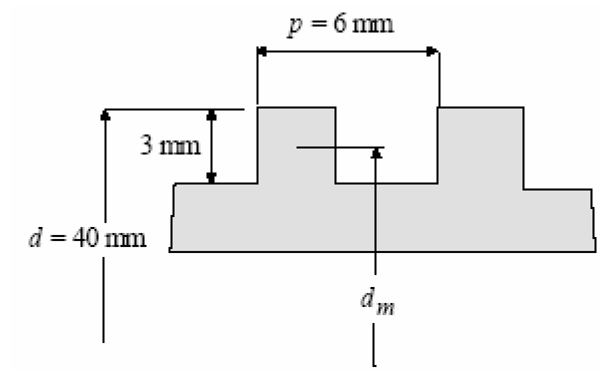
The problems are:		Enter problem number(s) that you selected
Machine Design	3 problems	( ) ( ) ( )
Kinematics & Dynamics	3 problems	( ) ( ) ( )
Controls & Vibration	3 problems	( ) ( ) ( )

The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your student ID number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

Machine Design #1

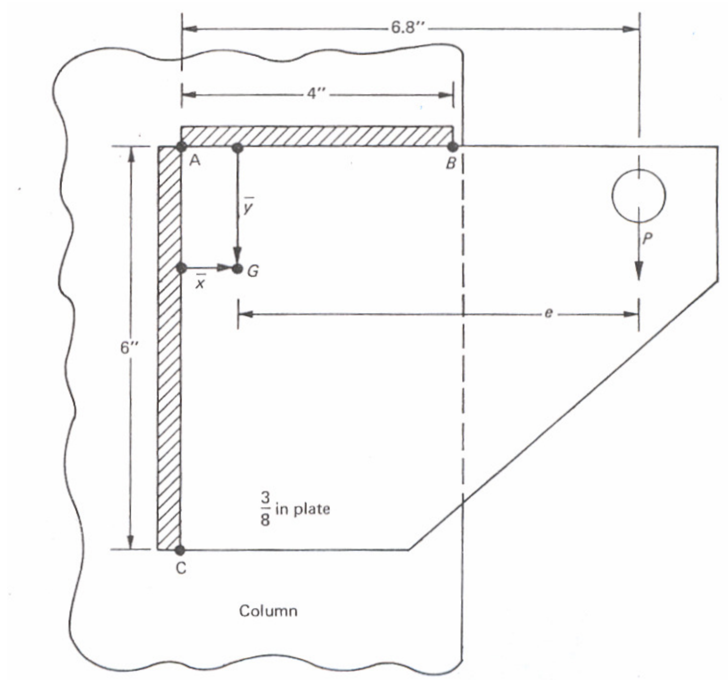
A 40 mm power screw having double square threads with a pitch of 6 mm is required to move a 10 kN load at a velocity of 48 mm/s. The coefficient of friction for the threads is 0.10 and for the collar it is 0.15. The frictional diameter for the collar is 60 mm. Determine the power required.





Machine Design #2

The bracket shown below is to be joined to a column by fillet welds. The dimensions of this weldment are as shown below. Find the required weld size if P = 2,000 lb, the allowable stress in the weld S_{sw} = 7,000 psi and the throat is 1 in.

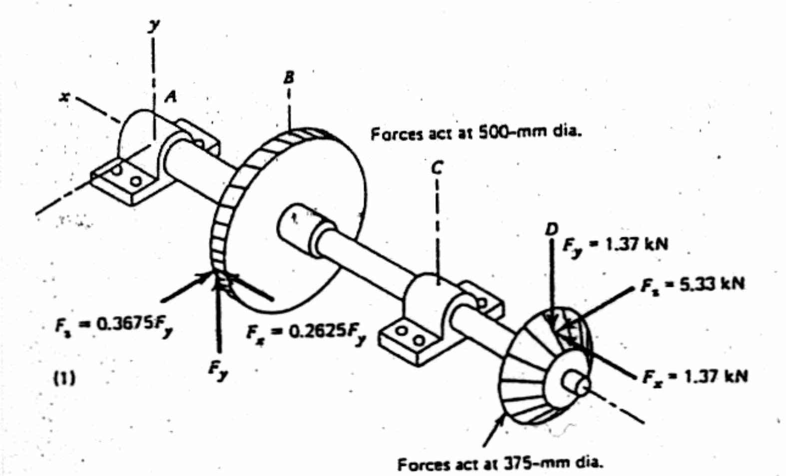


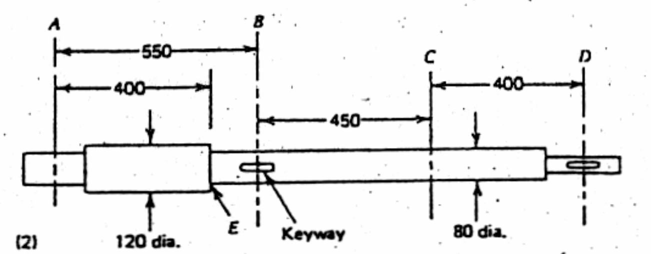
Show all of your work

Machine Design #3

The figure below shows a countershaft with helical gear (B), bevel gear (D) and two support bearings (A and C). The loads acting on the bevel gear are shown. The shaft is made of hardened steel with S_{ut} = 1069 Mpa and S_{yt} = 896 Mpa. All important surfaces are finished by grinding. All shoulder fillets have a radius of 5 mm.

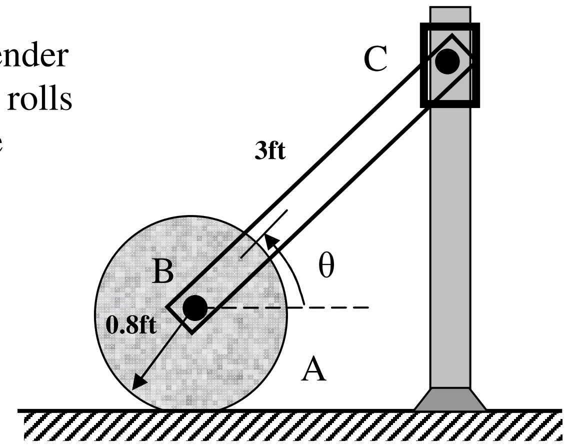
i. Determine the forces acting on the helical gear.
ii. The shaft has been designed so that only bearing A takes thrust. The bearing C is to take only radial load. At point B of the shaft, determine the FOS with respect to fatigue failure. If needed, assume a K_f = 1.6 for all fluctuating (bending, axial and torsional) stresses.





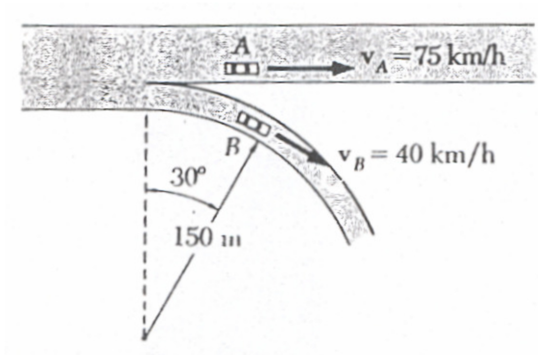
Kinematics & Dynamics #1

The system consists of a 20-lb disk A, 4-lb slender rod BC and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$, with the system release from rest when $\theta = 45°$.



Kinematics & Dynamics #2

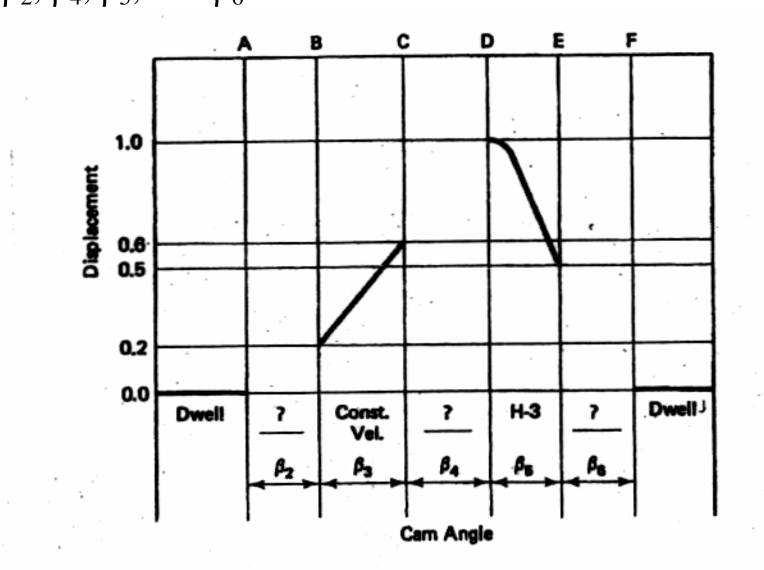
Automobile A is traveling along a straight highway, while B is moving along a circular exit ramp of 150-m radius. The speed of A is being increased at a rate of 1.5 m/s^2 and the speed of B is being decreased at a rate of 0.9 m/s^2. For the position shown, determine (a) the velocity of A relative to B, (b) the acceleration of A relative to B.



Show all of your work

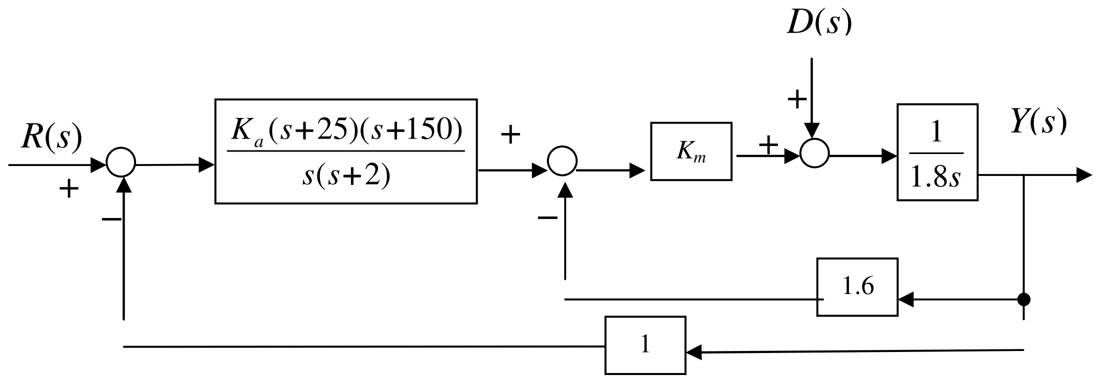
Kinematics & Dynamics #3

A roller follower is to move through a total displacement cycle shown below. Because of the operation performed by the mechanism, a portion of the rise motion (BC) must be at constant velocity. Using the follower displacement curve shown below, recommend what follower motion curves (harmonic, cycloidal etc.) should be used over segments AB, CD and EF of the motion and write the displacement *s* as a function of theta. Justify your selection of displacement curves. If the velocity over segment BC remains constant at 0.02 inch/deg and $\beta_3 = 20°$, find the value of active cam angle β_2, β_4, β_5, and β_6.



Controls & Vibration #1

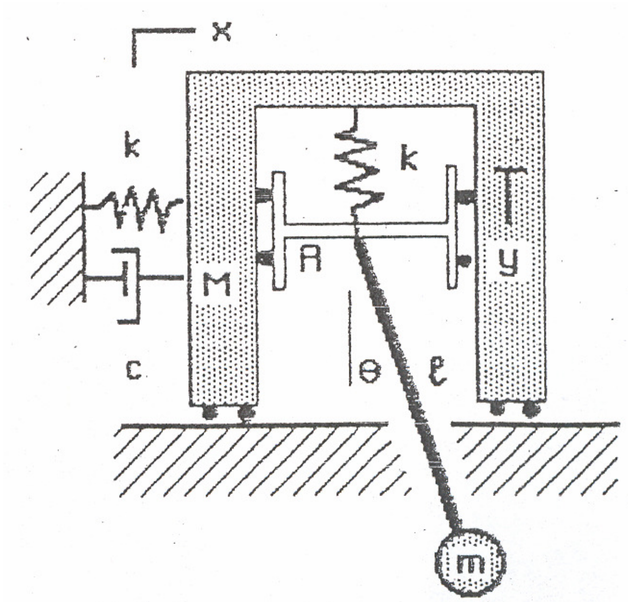
A precision speed control system is required for a platform used in gyroscope and inertial system testing where a variety of closely controlled speeds is necessary. The motor gain constant $K_m = 1.8$.



1) Determine the minimum loop gain necessary to satisfy the steady-state error requirement.
2) Determine the limiting value of gain *Ka* for stability using Routh-Hurwitz array method.

Controls & Vibration #2

Use Lagrange equations to derive the equation(s) of motion for the following system:



Let the mass of the slider A be considered negligible. Use x, y, and θ as the generalized coordinates. Let x and y be measured from the unstretched position of the respective springs.

Show all of your work

Controls & Vibration #3

A restaurant kitchen door is equipped with a torsional spring and a torsional damper so that it automatically returns to its closed position after being opened. The door has a mass of 60 kg and a centroidal moment of inertial about an axis parallel to the axis of door's rotation of 7.2 kg m^2. The torsional spring has a stiffness of 25 N.m/rad.

(i) What is the damping coefficient such that the system is critically damped?
(ii) A waiter with his arms full of dishes, but in a hurry, kicks the door causing it to open. What angular velocity must the kick impart to cause the door to open 70°.
(iii) How long after his kick will the door return to within 5° of completely closing?