MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part I
Mathematics and Fundamentals

Closed Book/Notes

September 10, 2015
Thursday
1:00 pm - 5:00 pm
EMS E250

The problems are:

		Enter problem number(s) that you selected	
Ordinary Differential Equations	2 problems	( )	( )
Partial Differential Equations	2 problems	( )	( )
Linear Algebra	2 problems	( )	( )
Calculus	2 problems	( )	( )
Numerical Methods	2 problems	( )	( )

The student should select 8 problems out of 10 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your assigned number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-F15/Fall 2015

Ordinary Differential Equation #1

Solve the differential equation:

$$y - xy' + e^{\frac{1}{x}} = 0$$

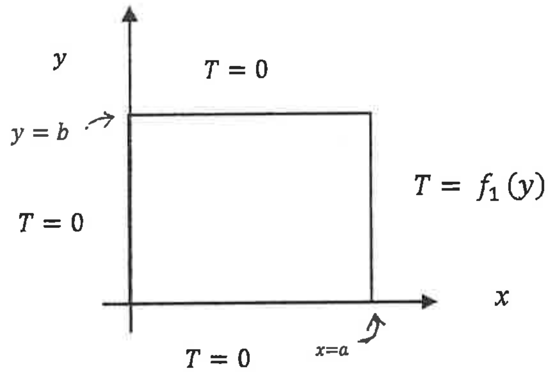
Ordinary Differential Equation #2

Solve the equation $$\mathbf{4\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0}$$

With the conditions of $y\ (2) = -5$ and $\frac{dy}{dt}\ (2) = 2$

Partial Differential Equation #1

Derive the solution $T\ (x, y)$ for the following 2-D Problem:



The governing differential equation is $\frac{\partial^2 T}{\partial\, x^2} + \frac{\partial^2 T}{\partial\, y^2} = 0$

Hint: The half-range since expansion in Fourier series is given as

$$f\ (x) = \Sigma_{n=1,2,3...}^{\infty} b_n\ Sin\ \frac{n\,\pi\,x}{L}$$

Such that

$$b_n = \frac{2}{L} \int_0^L f\ (x)\ Sin\ \frac{n\,\pi\,n}{L}\ dx$$

The half-range cosine expansion in Fourier series is given as

$$f\ (x) = a_o + \Sigma_{n=1,2,3...}^{\infty} a_n\ Cos\ \frac{n\,\pi\,x}{L}$$

Such that

$$a_o = \frac{1}{L} \int_0^L f(x)\ dx \quad \text{and} \qquad a_n = \frac{2}{L} \int_0^L f(x)\, Cos\ \frac{n\,\pi\,x}{L}\ dx$$

Partial Differential Equation #2

Solve, assume both of T_1 and T_2 are not zero.

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \qquad for\ 0 < x < L, \quad t > 0$$

$$u\,(0, t) = T_1, \qquad u\,(L, t) = T_2 \qquad for\ t > 0$$

$$u(x, 0) = f\,(x) for\ 0\ \leq x\ \leq L$$

Linear Algebra #1

1. Examine whether the matrix A is diagonalizable, where A is given by

$$(1)\ A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}, \qquad (2)\ A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

Linear Algebra #2

Consider the matrix A below is diagonalizable, find the $\lim_{k \to \infty} A^k$

$$Matrix\ A = \begin{bmatrix} -1/2 & -3 \\ 1/2 & 2 \end{bmatrix}$$

Calculus #1

Find the moment of inertia, about the y-axis, of the area enclosed by

$$r = a\,(1 - \cos\,\theta)$$

Consider the density of the plate to be 1.

Calculus #2

Evaluate the given integral:

$$\int x^2 \cos^2 x dx.$$

Numerical Methods #1

Write in the matrix form, perform Gauss Elimination and derive the LU decomposition

$$3\,x_1 - 0.1\,x_2 - 0.2\,x_3 = 7.85$$
$$0.1\,x_1 + 7\,x_2 - 0.3\,x_3 = -19.3$$
$$0.3\,x_1 - 0.2\,x_2 + 10\,x_3 = 71.4$$

Numerical Methods #2

Consider the following second-order method for the function *f*:

$$f^{n+1/2} = f^n + \frac{\Delta t}{2} L(f_i^{n+1/2})$$

$$f^{n+1} = f^n + \Delta t \ L(f_i^{n+1/2})$$

Where superscripts denote the time step and subscripts denote the space step. Also Δt is the time increment, and *L()* is the operator defined below.

Analyze for stability using the von Neumann method, for the following two cases.

(1)

$$L(f) = -\frac{(uf_{i+1} - uf_{i-1})}{2\Delta x}$$

where *u*=velocity=constant and Δx denotes the space increment.

(2)

$$L(f) = \alpha \frac{(f_{i+1} - 2f_i + f_{i-1})}{\Delta x^2}$$

where α=diffusion coefficient=constant.

MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Machine Design Stem

Open Book/Notes

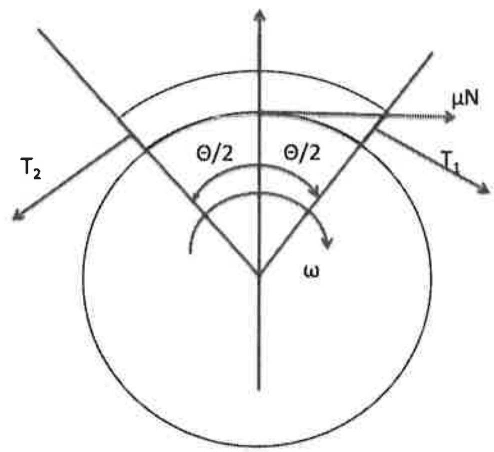
September 11, 2015
Friday
1:00 pm - 5:00 pm
EMS E250

The problems are:		Enter problem number(s) that you selected
Machine Design	3 problems	( ) ( ) ( )
Kinematics & Dynamics	3 problems	( ) ( ) ( )
Controls & Vibration	3 problems	( ) ( ) ( )

The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in (   ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your student ID number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

Machine Design #1

Consider a pulley with a flat belt

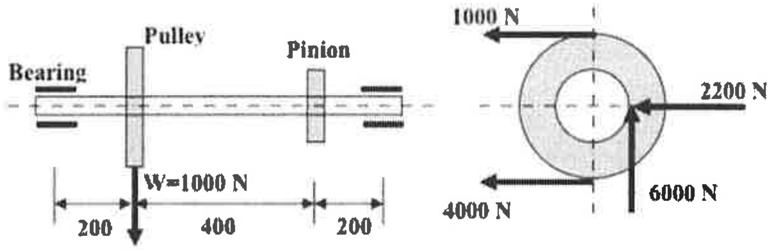


(a) Derive the relationship between the tensions $T_1, T_2 | T_1 > T_2$ of the belt given that the angle of contact is θ, the coefficient of friction is μ, the density per unit length of the belt is ρ and the radius of the pulley is r and ω is angular velocity in rad/sec.

(b) Given that the ultimate strength of the belt is σ and cross-section area is A, find the maximum torque that can be transmitted from the pulley to the belt for a given contact angle θ.

Machine Design #2

A circular shaft is to be designed to transmit is loaded as shown in the figure.



(a) Draw the shear force and bending moment diagram for both horizontal and vertical planes

(b) Assuming simple von-Mises criterion, and maximum allowable stress to be $\tau_{max} = Mpa$, find the diameter of the shaft. Assume the pulley to have a diameter of 400mm and the pinion to have a diameter of 100mm.

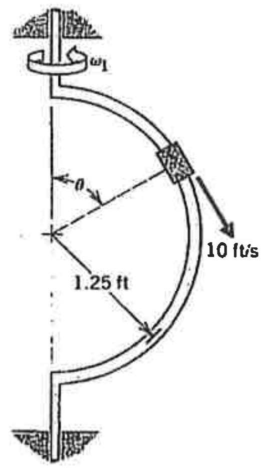
Machine Design #3

Given: A 50-mm-long, circular, hollow shaft made of carbon steel must carry a torque of 5000N·m at a maximum shear stress of 70 MPa. The inside diameter is 0.5 times the outside diameter.

Find: The outside diameter, the angle of twist, and the angular spring rate.

Kinematics & Dynamics #1

The collar slides along the curved bar at the constant rate of 10 ft/s. When the collar is at $\theta = 60^{\circ}$ the rotation about the vertical axis is $\omega_1 = 5\ rad/s$ and with $\dot{\omega}_1 =$ -2 $rad/s2$. Determine the corresponding acceleration of the collar.



Show all of your work

Kinematics & Dynamics #2

A roller follower is to move through a total displacement cycle with a single dwell during the cycle. Because of the operation performed by the mechanism, a portion of the rise must be at constant velocity. Using the displacement diagram shown below, an engineer has recommended the following displacement curves.

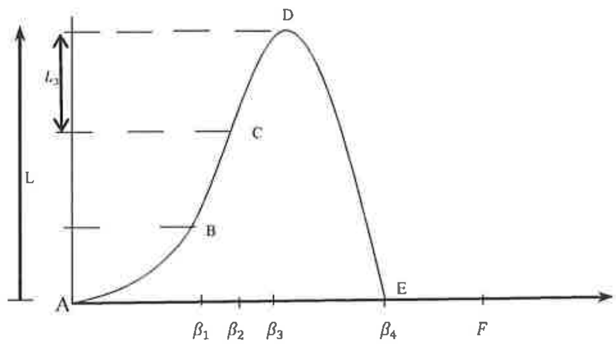
AB: Polynomial function

BC: Linear function: $s = k\theta \; \beta_1 \leq \theta \leq \beta_2$

CD: Harmonic: $s = \frac{L_3}{2}\left(1 - cos\frac{\pi\theta}{\beta_3}\right)$

DE: Cycloid: $s = L\left(1 - \frac{\theta}{\beta_4} - \frac{1}{\pi}sin\pi\frac{\theta}{\beta_4}\right)$

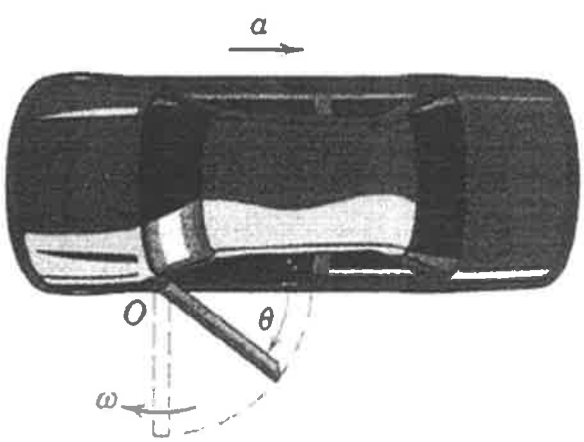
EF: Dwell



1. Are the above selected displacement functions suitable? Justify your answer. If the recommendations are not suitable, what recommendations will you make.
2. For the polynomial function over segment AB, what are the boundary conditions? What is the degree of the resulting polynomial function. **DO NOT DERIVE THE FUNCTION.**

Kinematics & Dynamics #3

A car door is inadvertently left slightly open when the brakes are applied to give the car a constant rearward acceleration a. Derive expressions for the angular velocity of the door as it swings pass the 90° position and the components of the hinge reactions for any value of θ. The mass of the door is m, its mass center is a distance $\bar{r}$ from the hinge axis O, and the radius of gyration about O is k_O.



Controls & Vibration #1

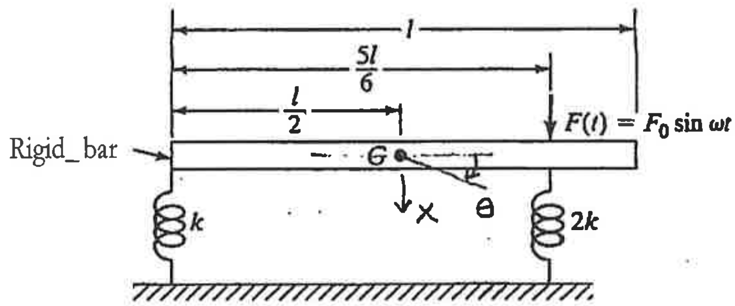
For the following non linear dynamic equations. u is the input. d is a constant.

$$\dot{x} = u \times sin(\sqrt{x}) + d$$

(a) Find the equilibrium equations
(b) Find the perturbed non-linear equation
(c) Linearize non-linear terms
(d) Find the linearized equations

Controls & Vibration #2

A uniform rigid bar of length l=1 m and mass m= 10 kg is supported on two springs and is subjected to a force as shown below. Derive the EOM of the bar for small displacements. For k= 1000 N/m, determine the natural frequencies and mode shapes for the bar.

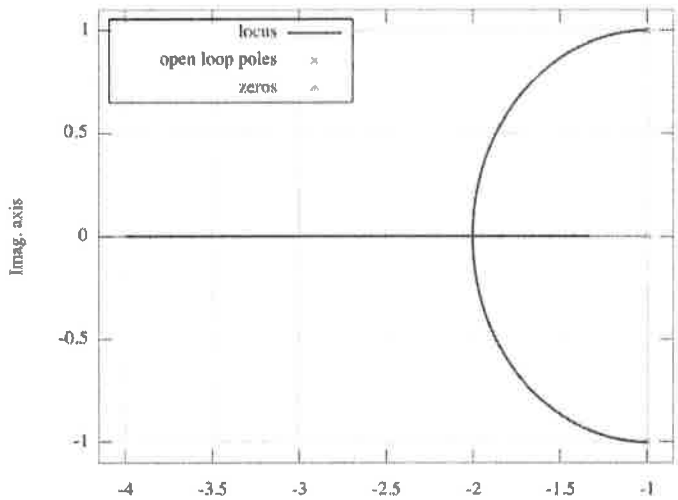


Controls & Vibration #3

From the root locus given below, find

(a) Find the loop transfer function G_L

(b) For a value of K at which the settling time is 2.8s find the rise time and damping ζ



MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Thermal Science Stem

Open Book/Notes

September 11, 2015
Friday
1:00 pm – 5:00 pm
EMS E250

The problems are:		Enter problem number(s) that you selected
Fluid Mechanics	3 problems	( ) ( ) ( )
Thermodynamics	3 problems	( ) ( ) ( )
Heat Transfer	3 problems	( ) ( ) ( )

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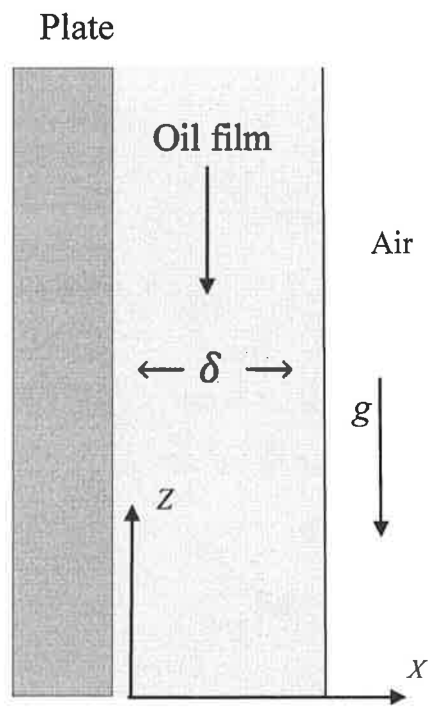
Fluid Mechanics #1

Oil of viscosity μ and density ρ drains steadily down the side of a tall vertical plate of width b into the paper. The width is large such that $\delta \ll b$. Far enough from the top of the film, both the film thickness δ and the shape of the velocity profile become independent of the distance z along the plate. Thus, the vertical velocity component w becomes a function only of x.

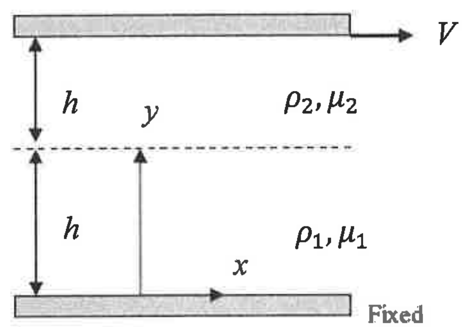


a) Assuming no slip at the plate, make a qualitative sketch of the velocity profile considering the boundary conditions at the wall and the film surface.

b) Suppose you are able to measure δ and the slope of the velocity profile at the wall $(dw/dx)_{wall}$. Find an expression for the viscosity of the liquid as a function of $\rho, \delta, (dw/dx)_{wall}, and\ g$. Show that your units are consistent.

Fluid Mechanics #2

Two immiscible liquids of equal thickness h are being sheared between a fixed and a moving plate, as shown below. Gravity is neglected, and there is no variation with x. Find an expression for (a) the velocity at the interface; and (b) the shear stress in each fluid. Assume steady laminar flow.



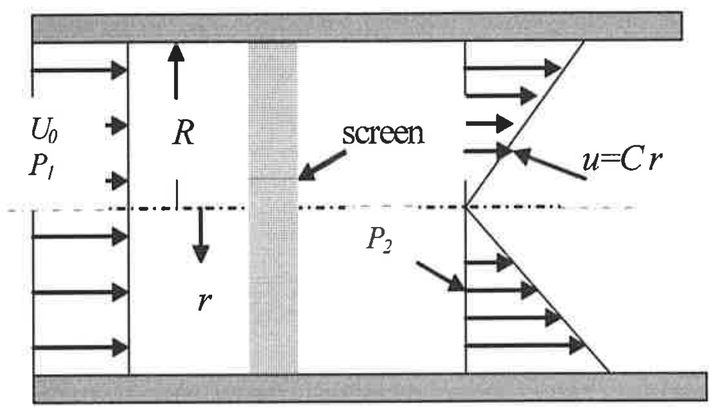
Fluid Mechanics #3

A uniform velocity profile airflow with density, ρ, enters the circular pipe at the velocity U_0. A variable mesh screen produces a linear and axially-symmetric velocity profile, $u=Cr$, as indicated in the figure (shown below) in the air flow through a radius R circular cross-sectional duct. The static pressure upstream and downstream of the screen are P_1 (at the entrance) and P_2 (at the section where the velocity profile becomes triangular shape), and are uniformly distributed over the flow cross-sectional area.

(1) Determine the velocity coefficient C in terms of U_0 and R.

(2) Obtain the drag force due to the screen and the friction in terms of P_1, P_2, ρ, R, and U_0.

HINT: Use the control volume approach.



Thermodynamics #1

Air enters a gas turbine at a temperature of 1200 K and a pressure of 1.2 MPa. The air exits the gas turbine at a pressure of 0.1 MPa. The isentropic efficiency of the turbine is 0.80, and the mass flow rate of the air is 50 kg/s. The air exiting the gas turbine is then directed through a heat exchanger that supplies some of the heat for water passing through a steam generator in a Rankine cycle. The air exits this heat exchanger at 350˚C.

The water enters the steam generator at 200˚C and 10 MPa, and exits at 10 MPa and 450˚C. The mass flow rate of the water is 40 kg/s. What is the rate of additional supplemental heating that is required for this water, after it has been heated in the heat exchanger described above?

Thermodynamics #2

Nitrogen at 60 psia and 300°F with a volume of 10 ft³ is allowed to expand in a polytropic process such that $Pv^n = Constant$. If n = 1.2 and the final volume is 50 ft³, determine (a) the work in Btu and (b) the heat transfer in Btu during this process. Show and reference all work.

Thermodynamics #3

A vapor power plant has an output power of 200MW. A steam at 180bar and 520°C enters the turbine of a vapor power plant. The pressure at the exit of the turbine is 0.08bar and liquid leaves the condenser at the same pressure with the saturated liquid. The liquid water is then pumped up to the pressure of 180bar. The turbine has an isentropic efficiency of 80% and the pump has 75%, respectively. For the cycle, compute:

(a) The net work per unit mass of steam flow
(b) The mass flow rate of the steam
(c) The total heat transfer of steam passing through the boiler
(d) The thermal efficiency
(e) The heat transfer to cooling water passing through the condenser.
(f) The mass flow rate of the condenser cooling water. The cooling water enters the condenser at 12°C and leaves it at 22°C.

Heat Transfer #1

A flow moves between two parallel plates. Assume the flow is laminar, incompressible, fully developed, and steady with constant properties. The heat is supplied through a constant het flux of q_0 from both plates.

(a) Obtain the expression of velocity profile in a fully developed region.

(b) Neglecting the viscous dissipation term, obtain the expression for the energy equation.

(c) Using appropriate non-dimensional parameters, rewrite the energy equation with the boundary conditions.

$$\theta = \frac{T - T}{q_0 w / k}; \quad \eta = \frac{x}{w}; \quad \varsigma = \frac{kz}{\rho C_p v_{max} w^2}$$

(d) Obtain the asymptotic solution of θ for large z.

Heat Transfer #2

80 ˚C Water is pumped through 100 m of stainless pipe (k=16 W/m K). The inner and outer radii of the pipe are 47 mm and 50 mm respectively. The heat transfer coefficient due to water is 2000 W/m² K. It is known that the outer surface of the pipe loses heat to 20 ˚C air by convection and the heat transfer coefficient is 200 W/m² K. Calculate the heat flow through the pipe. Also calculate the heat flow through the pipe when a layer of insulation (k=0.1 W/m K) and 50 mm radial thickness is wrapped around the pipe.

Heat Transfer #3

Boil-off from a Cryogenic Dewar Flask

Liquid oxygen is stored in a thin-walled spherical container, 96 cm in diameter, which is enclosed in a concentric container 100 cm in diameter. The surfaces facing each other are plated and have an emittance of 0.05, and the space in between is evacuated. The inner surface is at 95K and the outer surface is at 280K. (1) What is the oxygen boil-off rate? (2) If a thin radiation shield of emittance of 0.06 is placed midway between the containers, what is the new boil-off rate?