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MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part I
Mathematics and Fundamentals

Closed Book/Notes

February 4, 2016
Thursday
1:00 pm - 5:00 pm
EMS E250

The problems are:

		Enter problem number(s) that you selected	
Ordinary Differential Equations	2 problems	( )	( )
Partial Differential Equations	2 problems	( )	( )
Linear Algebra	2 problems	( )	( )
Calculus	2 problems	( )	( )
Numerical Methods	2 problems	( )	( )

The student should select 8 problems out of 10 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your assigned number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-S16/Spring 2016

Ordinary Differential Equation #1

Find the solution of

$$2xyy' = 3y^2 + x^2$$

With the boundary condition $y(1) = 2$

Ordinary Differential Equation #2

Find the general solution of the nonhomogeneous equation

$$y'' - y' - 2y = e^{3x}$$

where $y' = \frac{dy}{dx}$.

Partial Differential Equation #1

It is known that the following partial differential equation may be solved by direct integration:

$$\frac{\partial^2 U}{\partial x \partial y} = x^3 \, y$$

(a) Determine a generalized solution to the given PDE above

(b) Determine the particular solution for which $U\,(x, 0) = x^3$ and $U\,(2, y) = \sin y$.

Partial Differential Equation #2

For a homogeneous spherical solid with constant thermal diffusivity K and no heat sources, the equation of heat conduction becomes

$$\frac{\partial \Theta(r,t)}{\partial t} = K \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \Theta(r,t)}{\partial r}\right)$$

Using the method of separation of variables, find a general solution of this equation containing no more than 3 unknown constants (K is considered to be a known constant).

Hint. The following ODE $x^2 R''(x) + 2xR'(x) + a^2x^2R(x) = 0$ has a general solution $R(x) = Aj_0(\alpha x) + By_0(\alpha x)$, where j_0 and y_0 are spherical Bessel functions, and α is a constant.

Linear Algebra #1

Consider the planes $x_1 + x_2 + 3x_3 = 4$ and $x_1 + 2x_2 + 4x_3 = 5$ *in* $\mathbb{R}^3$. Find the parametric equations for the line of intersection of these two planes.

Linear Algebra #2

Find the eigenvalues and the corresponding normalized eigenvectors of the matrix.

$$A=\begin{bmatrix} -2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 6 \end{bmatrix}$$

Calculus #1

Consider the function $y(x) = x^3 e^x$. Find the area bounded by this curve, the x-axis, and the lines x=0 m and x=5m.

Calculus #2

What is $\int_C (xy + z^2)ds$, where C is the arc of the helix

$$x = cost, \quad y = \sin t, \quad z = t$$

which joins the points $(1 \quad 0 \quad 0)$ and $(-1 \quad 0 \quad \pi)$?

Numerical Methods #1

The least squares method chooses the functional form that minimizes the sum of the squares of the deviations. For example, given N data points, $[x_i, y(x_i)] = (x_i, y_i)$, the approximating function $\boldsymbol{y = a + bx}$ minimizes the following function:

$$s(a, b) = \sum_{i=1}^{N} (y_1 - a - bx_i)^2.$$

For the following data, determine a least squares straight line.

x	y
3	5
4	20
5	30
6	50
7	70
8	100

Numerical Methods #2

(1) By using Newton Raphson method, explain how to find the roots of the following equation:

$$x^3 = x^2 - 2x + 1$$

(2) Attempt to find a root up to thrcc steps. Start with the value x=0.

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MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Machine Design Stem

Open Book/Notes

February 5th, 2016
Friday
1:00 pm - 5:00 pm
EMS E250

The problems are:		Enter problem number(s) that you selected
Machine Design	3 problems	( ) ( ) ( )
Kinematics & Dynamics	3 problems	( ) ( ) ( )
Controls & Vibration	3 problems	( ) ( ) ( )

The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your student ID number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-S16/Spring 2016

Machine Design #1

Two forces are applied to pipe AB made of AISI 1010 HR steel (Fig. 1). Knowing that the pipe had inner and outer diameters of 30 mm and 36 mm respectively (i) draw the FBD of the pipe the forces, moments and torques acting at point (b) (ii) determine the FOS with respect to yielding at point (b)

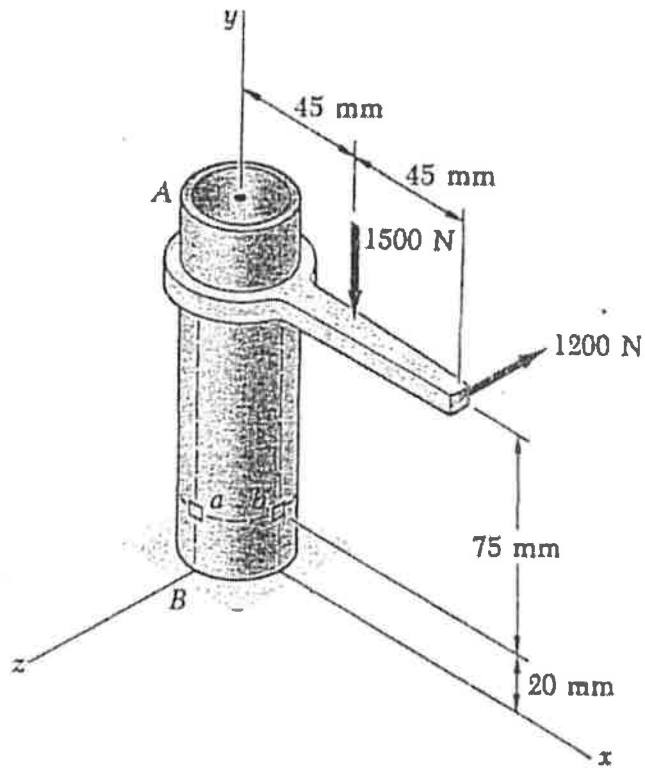
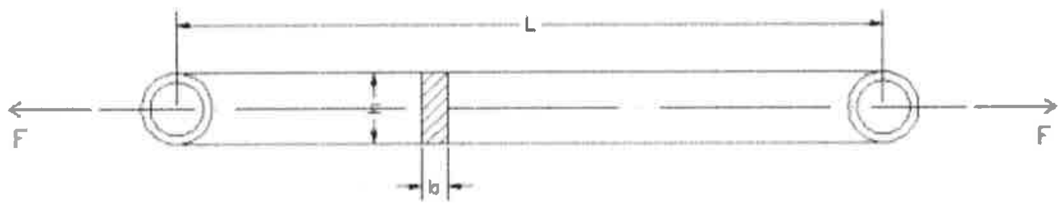


Fig. 1

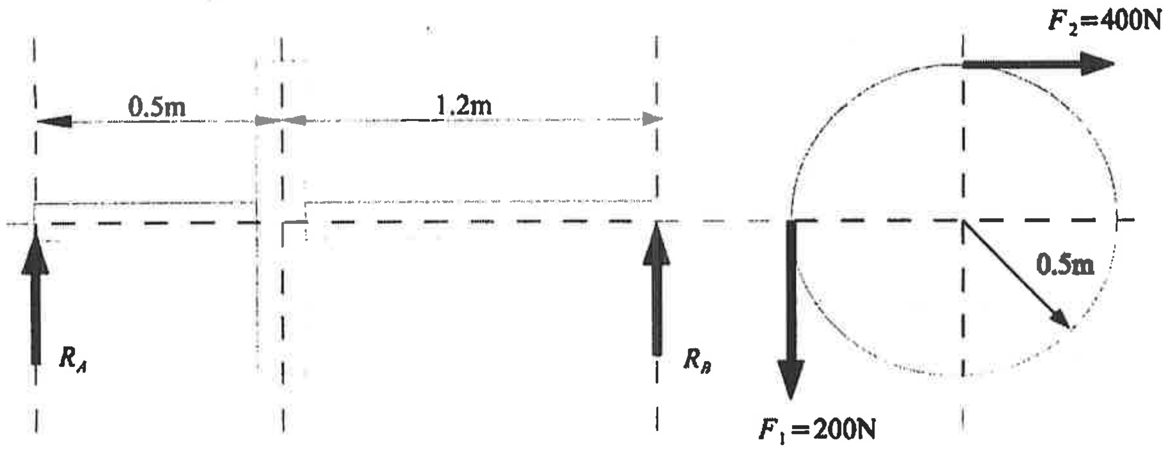
Machine Design #2

The link shown, made of AISI C1045 steel, as rolled, is subjected to a tensile load of 8000 lb. Let $h = 1.5b$. If the load is repeated but not reversed, determine the dimensions of the section with the design based on (a) ultimate strength, (b) yield strength. (c) If this link, which is 15in. long., must not elongate more than 0.005 in., what should be the dimensions of the cross section?



Machine Design #3

For the transmission shaft shown below:



Assume the following properties with usual notation

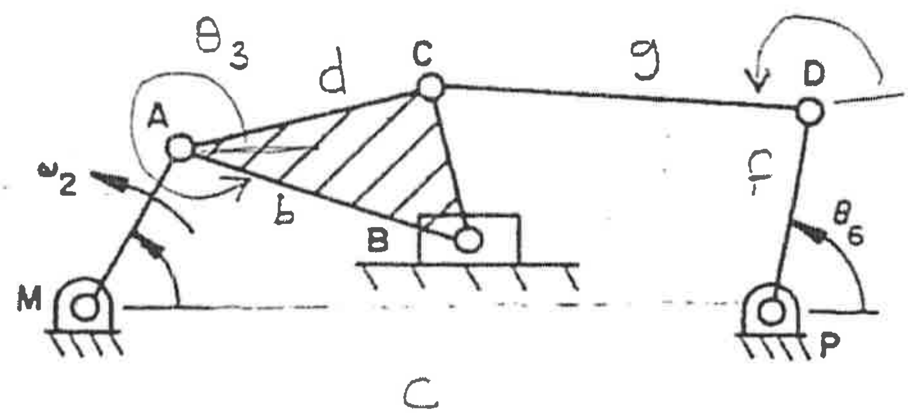
S_e	214 MPa
S_{ut}	241 Mpa
K_f	1.58
K_{fs}	1.39
n	2

Find the diameter d of the shaft using the DE-Goodman failure criterion (Hint: Here $M_a, T_a, T_m \neq 0,\ M_m = 0$

Kinematics & Dynamics #1

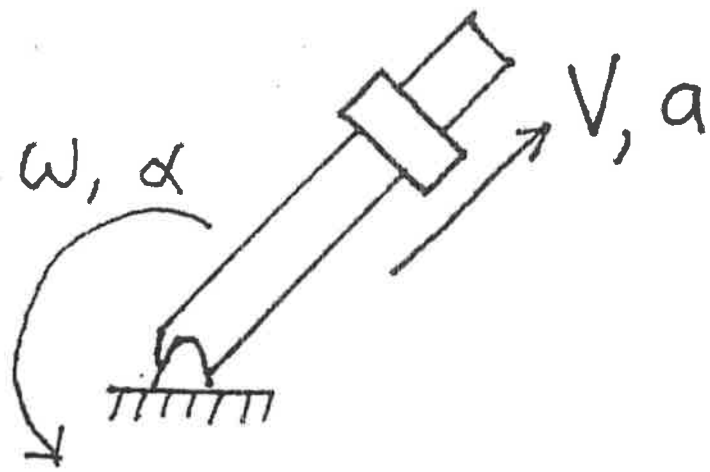
For the mechanism shown below, derive the loop closure equations. Choose input equal to DOF and identify the known and unknowns in these equations.

Given MA=1, AB=2.5, eccentricity e=0.75 in (vertical distance between M and B), $\theta_2 = 45$ *and* $\omega_2 = 5$ rad/sec, calculate the velocity of point B in one of the assembly modes. (Hint: Solve the displacement problem first analytically. You need to solve this problem for first loop M-A-B).



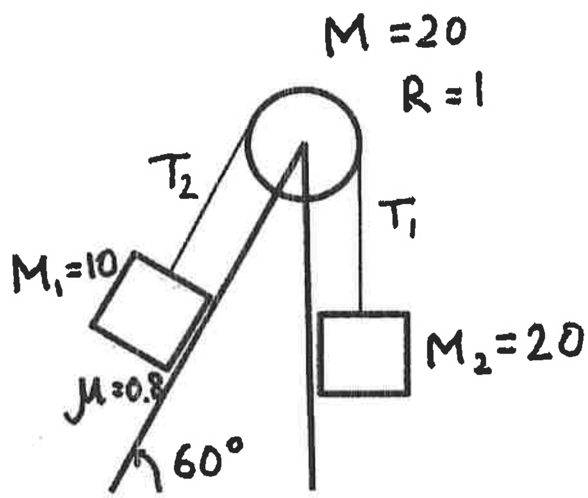
Kinematics & Dynamics #2

A link rotates with the angular velocity $\omega = 3\ rad/s$ and angular acceleration $\alpha = 1rad/s^2$. A slider of mass m=2kg slides along the link at the distance r=0.1 m with the relative velocity V=0.5 m/s and acceleration $\alpha = 1m/s^2$. The coefficient of friction between the slider and the link is $\mu = 0.3$. Calculate the friction force.



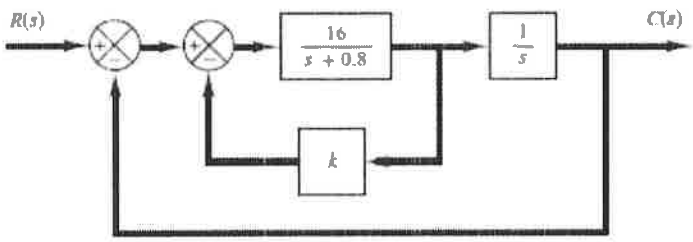
Kinematics & Dynamics #3

Two blocks ($M_1 = 10kg$ and $M_2 = 20kg$) and a solid disk ($M = 20kg,\ R = 1\ m$) are connected by a rope as shown. The coefficient of friction $\mu = 0.8$, slope angle $\alpha = 60^\circ$. Find acceleration of the system and tensions T_1 and T_2 in the rope.



Controls & Vibration #1

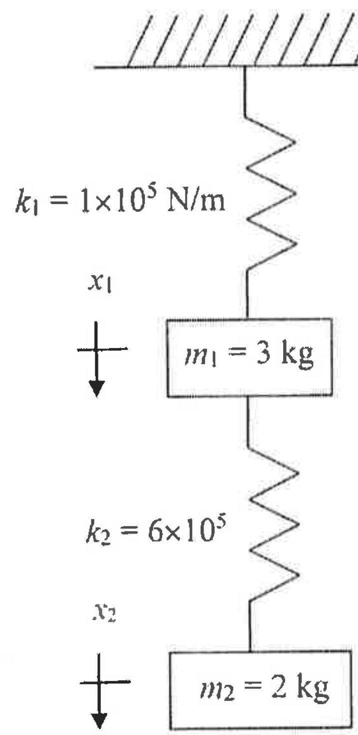
Consider the system shown in figure below, determine the value of k such that the damping ratio (ζ) is 0.5. Then obtain the rise time T_r, peak time T_p, maximum overshoot M_p, and settling time is T_s in the unit-step response. Find the range of k that keeps the system stable.



Controls & Vibration #2

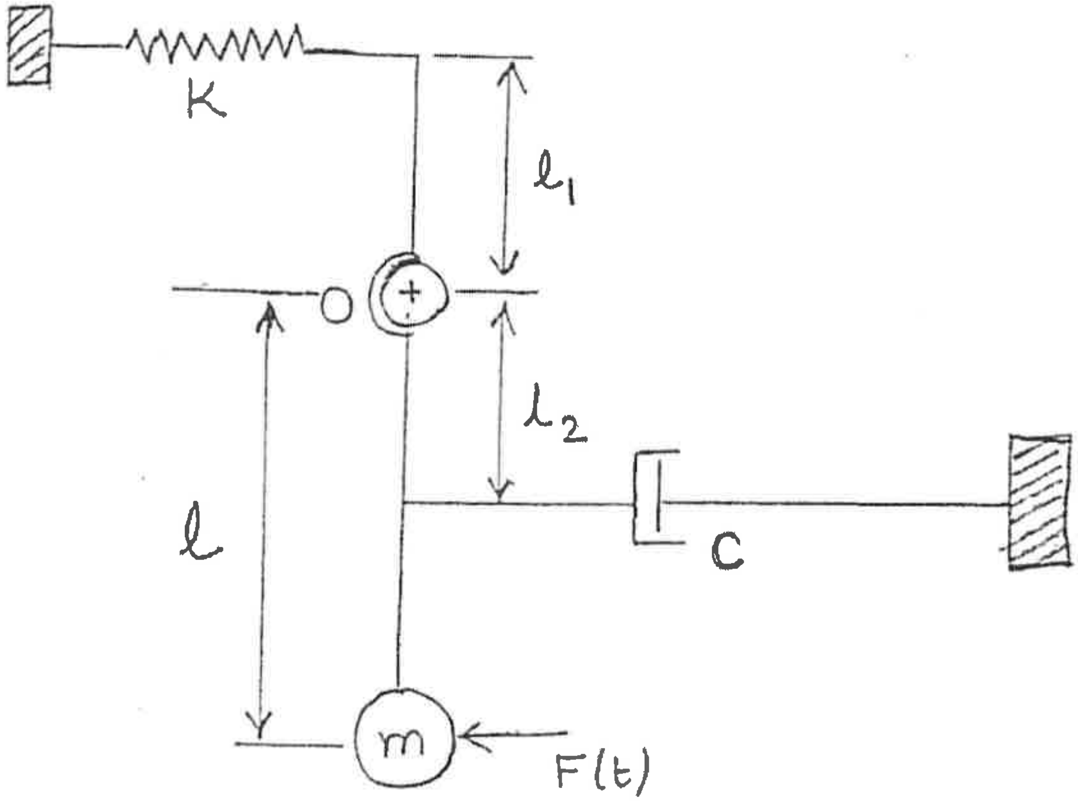
Given the two degree of freedom spring-mas system shown below, complete the following.

(a) Write the equations of motion in matrix form.
(b) Write the system characteristic equation using Laplace notation. Your solution should be a polynomial that is quadratic in S^2 with appropriate numerical coefficients.
(c) Calculate the natural frequencies (in Hz).
(d) Determine the two mode shapes (normalize to coordinate x_1).



Controls & Vibration #3

Consider the pendulum mechanism shown below which is used to model the brake pedal in an automobile. The pendulum is pivoted at point O. Derive the EOM governing this system. For k=4000 N/m, l_1=1.5m, l=1m, and m=40 kg, determine the damping constant c such that the damping ration of the system is 0.2. Also determine the amplitude of vibration of steady state response when F(t)= 10 cos(10t).



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MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Thermal Science Stem

Open Book/Notes

February 5th, 2016
Friday
1:00 pm – 5:00 pm
EMS E250

The problems are:		Enter problem number(s) that you selected
Fluid Mechanics	3 problems	( ) ( ) ( )
Thermodynamics	3 problems	( ) ( ) ( )
Heat Transfer	3 problems	( ) ( ) ( )

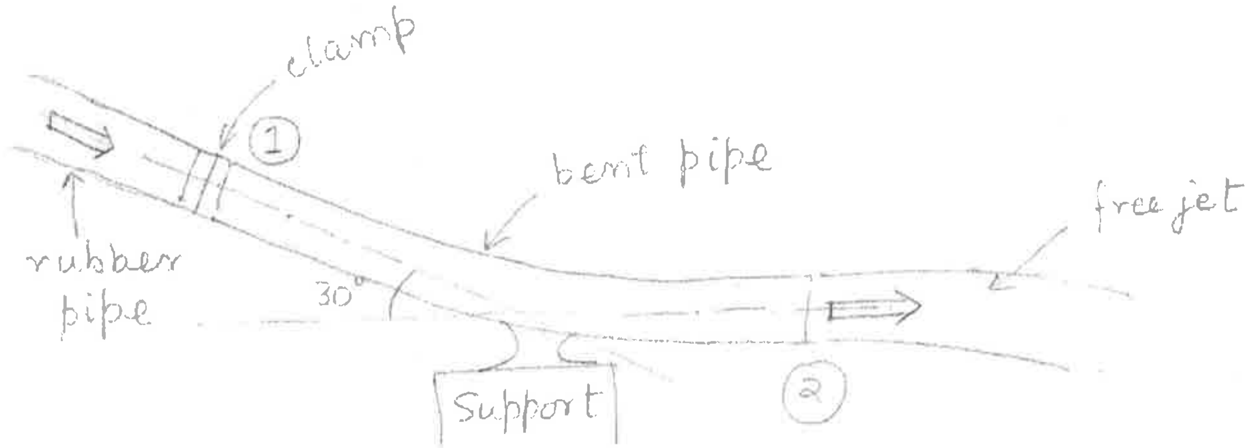
The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose **at least one problem from each category listed above**. Use one exam book (blue book) for each problem. Include your assigned number and **NOT** your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-S16/Spring 2016

Fluid Mechanics #1

Water is flowing through a bent pipe shown below at the rate of 1 kg/s. If the inner pipe diameter is 2 cm and no force is being transmitted through the rubber pipe, do the following parts: (a) If the head loss due to friction inside the pipe is 10 m, find the gage pressure at the inlet 1. (b) Estimate the horizontal and vertical components of force provided by the support to keep the pipe fixed in its place. (Sketch the support force vector.) Take the density of water to be $10^3\ kg/m^3$ and the acceleration due to gravity to be $10\ m/s^2$.

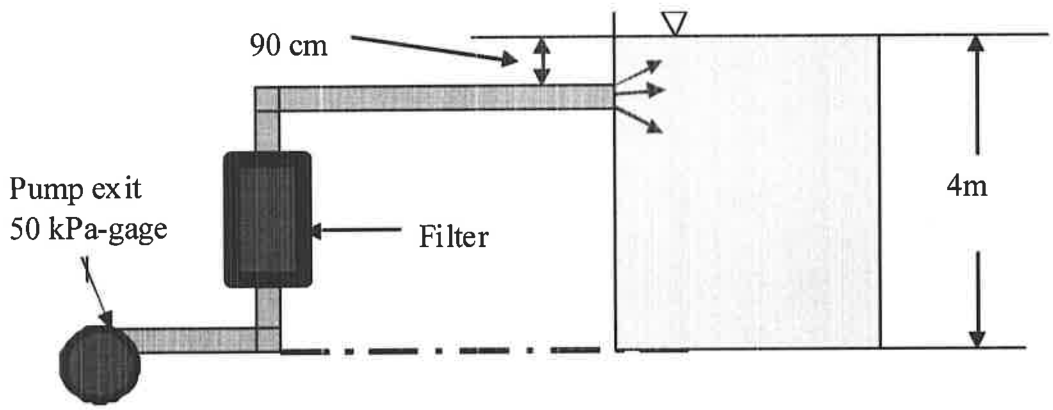
Make suitable assumptions. Show all your steps and diagrams – you will be graded for your work too.



Fluid Mechanics #2

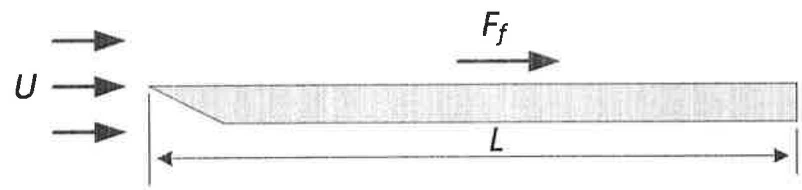
The water pump produces 50 kPa-gage at 10˚C at the discharge point as shown in the figure below. The pipe system includes one filter (K=7) and two regular screwed 90-degree elbows (K=0.5 each). The pipe is a 50 m-long in total and 10 cm-diameter commercial steel pipe.

(1) What is the loss coefficient at the exit of the pipe?
(2) Calculate the average velocity through the pipe.
(3) Calculate the volumetric flow rate.



<u>**Fluid Mechanics #3**</u>

(a) Find the friction drag on one side of a smooth flat plate of 150 mm wide and 500 mm long, placed longitudinally in a stream of crude oil (s = 0.925) at 20 ˚C flowing with undisturbed velocity of 600 mm/s. See the figure. (b) Find the thickness of the boundary layer and the shear stress at the trailing edge of the plate.



Thermodynamics #1

Tap water enters an ice machine at 55°F and leaves as ice at 32°F. If the COP of the ice machine is 3.0 and it takes 150 Btu of energy to be removed from each *l*bm of water at 55°F to turn it into ice at 32°F during this operation, determine (a) the required power input in hp for an ice production rate of 20 *l*bm/h, (b) the heat exhausted into the 75°F kitchen in Btu/h, and (c) the maximum COP if the $\overline{T_C} = (55 + 32°F)/2$. Show and reference all work.

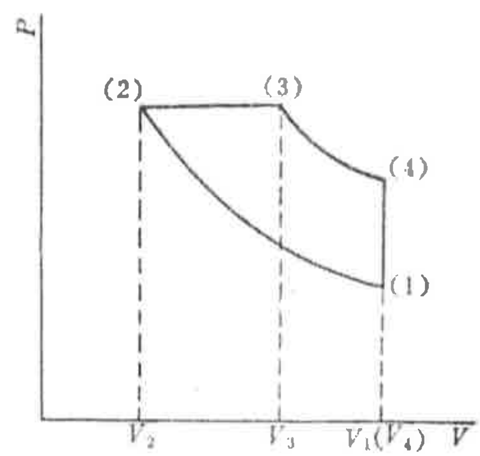
Thermodynamics #2

An ideal gas goes through the following processes

(1) Adiabatic compression from V_1, T_1 *to* V_2, T_2.
(2) Constant pressure expansion (absorbing heat) from V_2, T_2 *to* V_3, T_3.
(3) Adiabatic expansion from V_3, T_3 *to* V_1, T_4.
(4) Constant volume change from V_1, T_4 *to* V_1, T_1.

Prove the efficiency for the thermal cycle is

$$\eta = 1 - \frac{\rho^{\gamma-1}}{\gamma\varphi^{\gamma-1}(\rho-1)} \quad \text{where } \rho = \frac{V_3}{V_2}, \quad \varphi = \frac{V_1}{V_2}, \quad \gamma = \frac{C_p}{C_v}$$



Thermodynamics #3

Air enters the compressor of an ideal air-standard Brayton cycle at 1.01bar, 310K. The compressor pressure ratio is 12 and the temperature at the turbine exit is 770K. The velocity at the inlet of the compressor is 10m/s and the cross sectional area of compressor inlet is $0.5m^2$. Determine:

(a) The mass flow rate in kg/s.
(b) The net power developed in kW.
(c) The thermal efficiency
(d) The back work ratio

Use "Air-standard Analysis", which means the specific heats are treated as a variable.

[Hint] The cycle is a steady state with negligible kinetic and potential energy changes across each component.

Heat Transfer #1

A cross-flow heat exchanger with both fluids unmixed is to heat water with hot exhaust gas. The water ($C_{p,\ w} = 4400\ J/(\text{kg}\cdot{}^\circ\text{C})$) enters the tubes at 25°C at a rate of 1 kg/s, while the exhaust gas ($C_{p,\ g} = 1100\ J/(\text{kg}\cdot{}^\circ\text{C})$) enters the exchanger at 200°C at a rate of 2 kg/s. If the total heat transfer surface area is $A = 36.67\ m^2$ and the overall heat transfer coefficient is $U = 120\ W/(m^2\cdot{}^\circ\text{C})$, determine (a) the heat transfer rate in Watts, (b) the outlet exhaust gas temperature in °C, and (c) the outlet water temperature in °C. Show and reference all work.

Heat Transfer #2

Laminar flow moves in an annulus inner and outer radii βR and R, respectively. Heat is added to the fluid through the inner cylinder wall at a rate of $\dot{q}$" (heat per unit area per unit time), and the outer cylinder wall is thermally insulated. Let U_{max} = maximum velocity in the pipe, and the initial temperature is T_1.

(1) Set up the dimensionless equations for the governing equation.
(2) Set up the boundary and initial conditions.
(3) Determine the expression for the dimensionless temperature. You don't have to obtain the coefficient in the expression, but explain how to get it.

[Hint] $\theta = k(T - T_1) \;/\; \dot{q}''R$; $\xi = r/R$; $\eta = kz/(\rho C_p U_{max} R^2)$

Heat Transfer #3

The surface of a 10-m-long, 10-cm-diameter horizontal cylinder of dull brass ($\varepsilon = 0.22$) is maintained at 320°C. The cylinder is located within a large black warehouse maintained at 20°C. If the cylinder experiences a cross-flow of 20°C air at a velocity of 1 m/s due to the ventilation, determine (a) the heat loss due to forced convection in Watts, (b) the heat loss due to radiation in Watts, and (c) the total heat loss from the cylinder in kW. Show and reference all work.