The Stratification of Moduli Spaces of Algebras by Projective Orbifolds

Low dimensional moduli spaces of complex Lie and associative algebras have a natural stratification by projective orbifolds of a very simple type. They are given by complex projective spaces $\mathbb{C}P^n$, with the action of a subgroup of the symmetric group $\Sigma_{[n+1]}$, acting in the natural manner by permuting the projective coordinates. For Lie algebras, a part of the moduli space is given by a stratification of the space of $nxn$ complex matrices acted on by $\text{GL}(n,\mathbb{C}) \times \mathbb{C}^*$, where the $\text{GL}(n,\mathbb{C})$ acts by conjugation and the $\mathbb{C}^*$ by multiplication. This space is closely related to the moduli space of similarity classes of matrices, which has a description related to the Jordan decomposition of matrices. This moduli space is completely determined by a stratification by projective orbifolds of the type given above, and this stratification is the only stratification completely compatible with the deformation theory of the matrices. The same is true for the stratifications of the moduli spaces of algebras. In this talk, I will discuss a conjecture related to the stratification of such moduli spaces of algebras.