MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part I
Mathematics and Fundamentals

Closed Book/Notes

November 15, 2007
Thursday
1:00 pm - 5:00 pm

The problems are: Enter problem number(s) that you selected

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The student should select 8 problems out of 10 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose at least one problem from each category listed above. Use one exam book (blue book) for each problem. Include your assigned number and NOT your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-F07/Fall 2007
**Ordinary Differential Equation #1**

1) Find the solution $y(t)$ of the initial value problem

$$y'' + y = 0, \quad y(0) = 2, \quad y'(0) = -3$$

2) What are the maximum and minimum values of the solution $y(t)$ for $-\infty < t < \infty$?
Ordinary Differential Equation #2

Consider a damped spring-mass system. The position from equilibrium, $y$, at a time, $t$, is described by the equation

$$\frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + 4y = 0$$

The momentum for the system is given by $p = \frac{dy}{dt}$

For the initial conditions of $y(0) = 4$, $p(0) = -5$, find
(a) the equation describing the position, $y$, as a function of time,
(b) the equation describing the momentum, $p$, as a function of time, and
(c) the total energy of the system at $t = 3$. 
Partial Differential Equation #1

The differential equation to describe the transient heat conduction is given as:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0
\]

with the boundary conditions of \( T \) as:

**B.C.**:

\[
\begin{align*}
\frac{\partial T}{\partial x} &= 0 & \text{at } x = 0 \\
\frac{\partial T}{\partial x} + HT &= 0 & \text{at } x = a \\
T &= f(x) & \text{at } y = 0 \\
\frac{\partial T}{\partial y} &= 0 & \text{at } y = b
\end{align*}
\]

where \( H \) is the convective heat transfer, which you can treat as a constant.

Obtain an expression for the steady state temperature distribution \( T(x,y) \) in a rectangular region \( 0 \leq x \leq a, \quad 0 \leq y \leq b \).
Partial Differential Equation #2

Consider a semi-infinite rod of rectangular cross section \(2L \times 2\ell\) as shown in the figure below. The base temperature of the rod is \(\theta_0\), and the ambient temperature is 0. The heat transfer coefficient is large. Determine the steady state temperature of the rod using separation of variables. The governing equation can be described as follows:

\[
\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = 0
\]

The boundary conditions are:

\[
\frac{\partial \theta(0, y, z)}{\partial x} = 0, \quad \theta(L, y, z) = 0
\]
\[
\frac{\partial \theta(x, 0, z)}{\partial y} = 0, \quad \theta(x, \ell, z) = 0
\]
\[
\theta(x, y, 0) = \theta_0, \quad \theta(x, y, \infty) = 0 \text{ (finite)}
\]
**Linear Algebra #1**

Let $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear mapping defined by

$$F(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t)$$

Find a basis and the dimension of the image of $U$ of $F$

**Show all of your work.**
Linear Algebra #2

Determine the equation of the circle passing through the following three points in the plane (2,6), (6,4), and (7,1).
Calculus #1

A fluid flows out of a region bounded by the curve $6x^2 + 4y^2 = 24$. The velocity field of the fluid is defined by

$$F(x,y) = (3y + y^2 + 8x) \mathbf{i} + (x^4 - x + 6y - 3) \mathbf{j}.$$ 

Find the rate of flow out of the region. The velocity is measured in cm/s, and the area is measured in cm$^2$. 

Calculus #2

If \( \theta = t^n e^{-r^2/(4t)} \), what value of n will make

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}.
\]
Consider the typical diffusion equation:

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}
\]

Prove that when the following explicit scheme is used, the spatial order of accuracy is second order.

\[
u_{i,j+1} = ru_{i-1,j} + (1-2r)u_{i,j} + ru_{i+1,j} \quad \text{where} \quad r = \frac{k}{h^2}
\]

Here, \( h \) and \( k \) are spatial and temporal grid size. What value of \( r \) should be assigned if 4\(^{th}\) order spatial accuracy is required?
**Numerical Methods #2**

Using the second-order terms of the Taylor series expansion and the forward-time and centered-space differencing, show that the advection equation

\[
\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0
\]

(1)
can be formulated as:

\[
f_{i}^{n+1} = f_{i}^{n} - \frac{c}{2}(f_{i+1}^{n} - f_{i-1}^{n}) + \frac{c^2}{2}(f_{i+1}^{n} - 2f_{i}^{n} + f_{i-1}^{n})
\]

where

\[
c = \frac{u\Delta t}{\Delta x}
\]

What is the order of the accuracy?

Using von Neumann stability analysis, determine the stability condition.
MECHANICAL ENGINEERING DEPARTMENT

Ph.D. Qualifying Exam

Part II
Area of Concentration
Thermal Science Stem

Open Book/Notes

November 16, 2007
Friday
1:00 pm – 5:00 pm

The problems are:

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<td>Heat Transfer</td>
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Enter problem number(s) that you selected

The student should select 6 problems out of 9 problems given here. Write the problem numbers that you selected in ( ) above. Also circle the problems that you choose on each problem sheet. Choose at least one problem from each category listed above. Use one exam book (blue book) for each problem. Include your assigned number and NOT your name on each book. Submit both exam books and this problem sheet when you leave.

QualExam-F07/Fall 2007
**Fluid Mechanics #1**

A cylindrical annular flow with the inner cylinder rotating at a constant speed $\omega$ and the outer cylinder stationary is shown in the figure below. The radii of the outer and inner cylinders are $R_o$ and $R_i$, respectively.

1. Express the velocity along the wall in terms of the dimensions, rotating speed $\omega$ and the fluid properties, viscosity $\mu$ and density $\rho$.
2. What is the power needed to move per unit depth? Assume $R_o - R_i \ll R_o$.
3. For SAE 30 oil at 20°C, with $R_o = 10\text{cm}$, $R_i = 9\text{cm}$, and $\omega = 900\text{rpm}$, calculate the power.

[Hint] The equi-dimensional ordinary differential equation of the function $f$:

$$f'' + \frac{1}{r}f' - \frac{1}{r^2}f = 0$$

has a form of the general solution in the form of $f_1 = C_1 r$ and $f_2 = C_2/r$. 

![Diagram of cylindrical annular flow with inner cylinder rotating and outer cylinder stationary.](image-url)
Fluid Mechanics #2

A cylindrical rod rests on a wall of 45°. A gate is mounted on the left top side of the rod to hold water. Compute the horizontal and vertical forces due to the water acting on the 6-ft-diameter cylindrical rod per unit width (1-ft).
Fluid Mechanics #3

A flow field is given by $v = 2x^2 \hat{i} - 2yz \hat{j} - (y^2 + 3)\hat{k}$

- Show that the flow field is irrotational.
- Derive an expression for the velocity potential $\Phi$ within an arbitrary constant.
- If $\xi$ is a rather complicated path from $(0, 0, 0)$ to $(0, 0, 4)$, use $\Phi$ to compute the integral $I = \int \vec{v} \cdot d\vec{R}$ over $\xi$ where $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of a point on the curve.
- What is the value of $I$ if the integration is performed over a closed path? Do not repeat the calculations performed above to get to the answer. Hint: use a well known theorem.
**Thermodynamics #1**

Refrigerant 134a enters an insulated compressor operating at steady state as saturated vapor at -10°C at a rate of 18 kg/min. At the compressor exit, the pressure of the refrigerant is 5 bar. Kinetic and potential energy effects can be neglected. If the rate of entropy production must be kept less than 0.4842 kJ/k·min, determine the possible range of values for isentropic compressor efficiency.
Thermodynamics #2

A turbojet engine operates between pressure limits of 5 and 50 psia. The inlet air temperature to the compressor is -40°F and the upper temperature limit for the engine is 2000°F. Calculate the thrust for 1 lbm/sec of air flow, assuming isentropic compression and expansion and an inlet velocity of 300 ft/sec. Assume constant specific heats. Also calculate the heat input per pound mass of air.
**Thermodynamics #3**

A liquid-vapor mixture of water with a mass of 0.3 kg, at 150°C occupies an insulated tank with a volume of 0.1 m$^3$. Water vapor (and vapor alone) is removed from the tank until the temperature is 140°C. Determine the amount of water that is removed during the process.
Heat Transfer #1

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere. The properties of the junction are $k=35 \text{ W/m}^{-\circ\text{C}}$, $\rho=8500 \text{ kg/m}^3$, and $C_p=320 \text{ J/kg}^{-\circ\text{C}}$, and the convection heat transfer coefficient between the junction and the gas is $h=210 \text{ W/m}^2^{-\circ\text{C}}$. Determine how long it will take for the thermocouple to read 99% of the initial temperature difference.
Heat Transfer #2

A liquid metal flowing with uniform velocity $V$ through a pipe of periphery $P$ and cross section $A$ is electrically heated at the rate of $u''$ over a length of $\ell$ of the pipe. The inlet temperature of the liquid is equal to the ambient temperature $T_\infty$. The outside heat transfer coefficient is $h$. Neglecting the effect of axial conduction, and you should use the lumped temperature across the cross-sectional area of the pipe for part (a).

(a) Find the **steady** axial temperature distribution of the liquid metal.

(b) The flow of the liquid metal is suddenly stopped because of a pump failure. Find the **unsteady** temperature of the liquid metal. Hint: treat entire heating region as a single control volume.
A UWM Professor is working in his office that is maintained at an air temperature of 15°C. The room walls, floor, and ceiling are painted a dark gray (ε = 1.0) and are maintained at 15°C. Determine the heating load required by UWM to maintain the Professor’s body temperature at 25°C if he has an emissivity of ε_{Professor} = 1.0. Assume the Professor can be modeled as a vertical cylinder with a diameter of 0.3 m and a height of 2 m. Neglect end effects.
The problems are:

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<td>Controls &amp; Vibration</td>
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Machine Design #1

The figure fellow shows a vehicular jack which is to be designed to support a maximum load of 5000 Newtons. The link angle $\theta$ varies from 15 to 70 deg. The links are 300 mm long. Using an assumed FOS, and a suitable cross-section for the links, determine the required standard size for the links.
Machine Design #2

The figure below shows portion of a pump that is gear driven at uniform load and speed. The shaft is made of AISI 1045 cold-drawn steel and is supported by bearings mounted in the pump housing. The forces acting on the gear are shown below. Find the factor of safety with respect to fatigue failure at the fillet.
Machine Design #3

For the C-clamp shown below, the operator can comfortably exert a 20 lb force at the end of the handle, which creates a clamping force of $W = 1000$ lb. The screw has American Standard Threads ($2\theta = 60^\circ$) with 13 threads per inch (single-threaded). The outside diameter is 0.50 inch. The root diameter and root area are 0.4001 inch and 0.1257 inch$^2$, respectively. Knowing that $d_c = 0.50$ inch, $\mu_t = 0.12$, and $\mu_c = 0.25$, determine the required handle length $L$. 

![Diagram of C-clamp](image-url)
**Kinematics & Dynamics #1**

The rotations of an airplane about a set of xyz axes attached to the airplane are termed the roll rate $w_x$, the pitching rate $w_y$, and the yaw rate $w_z$. A passenger on a jumbo jet 30 m behind and 2 m above the center of mass $G$ of the airplane is walking at a constant rate of 1 m/sec. Determine the difference between the acceleration that this passenger experiences and the acceleration that the center of mass of the airplane undergoes, if the roll rate is 0.5 rad/sec, the pitching rate is -0.3 rad/sec, and the yaw rate is 0.1 rad/sec. All rotation rates are constant.

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Show all of your work.
The two bars are released from rest at the position $\theta$. Determine their angular velocities at the instant they become horizontal. Neglect the mass of the roller at $C$. Each bar has a mass $m$ and length $L$. 

![Diagram of two bars with a given angle $\theta$ and length $L$.]
Kinematics & Dynamics #3

For the 1-DOF mechanism shown below:

(a) Derive the loop equations and identify the known and unknowns in these equations.
(b) Derive the velocity equations and identify the known and unknowns in these equations.
(c) Derive the acceleration equations. Identify the known and unknowns in these equations.
Driverless vehicles can be used in warehouses, and many other applications. These vehicles follow a wire imbedded in the floor and adjust the steerable front wheels in order to maintain proper direction, as shown in Figure (a) below. The sensing coils mounted on the front wheel assembly detect an error in the direction of travel and adjust the steering. The block diagram for this system is shown on Figure (b).

For the system described above, design a controller such that

i) The steady state error ($e_{ss}$) to a unit ramp must be finite

ii) The natural frequency ($\omega_n$) is $\sqrt{5}$ rad/sec

iii) The damped natural frequency ($\omega_d$) is 2 rad/sec
Controls & Vibration #2

Consider the following closed-loop control system:

1) Determine the system type with respect to the disturbance input $D(s)$.
2) Determine the sensitivity of the closed-loop transfer function $T(s)$ from $D(s)$ to $Y(s)$ to changes in the parameter $A$, i.e. $S^{T(s)}_A$.
3) Let $D(s) = 0$, determine the steady state error $e_{SS}$ for a unit ramp reference input, i.e. $R(s) = \frac{1}{s^2}$.
4) Let $D(s) = 0$, determine the sensitivity of the steady state error $e_{SS}$ for unit ramp reference input with respect to changes in $K$, i.e. $S^{e_{SS}}_A$. 

![Closed-loop control system diagram]
Controls & Vibration #3

Find the natural frequencies and modes shapes of the vibrating system shown in the figure below. The following data applies: \( m_1 = 1 \text{ kg}, m_2 = 2 \text{ kg}, m_3 = 3 \text{ kg}, k_1 = 300 \text{ N/m}, k_2 = 500 \text{ N/m}, k_3 = 200 \text{ N/m}, I = 0.75 \text{ m}. \)